SOME MODELS FOR MANPOWER PLANNING
IN THE
INDONESIAN AIR FORCE

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THESIS

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IN THE
INDONESIAN AIR FORCE

by

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(20. ABSTRACT Continued)

condition, this study is intended to develop some mathematical models to assist the manpower planner in establishing a reasonable promotion policy which can create and maintain a well-balanced force structure in the Indonesian Air Force.

Some Models for Manpower Planning in the Indonesian Air Force

by

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ABSTRACT

The problem of determining a promotion policy which can maintain a well-balanced force structure, has long been a great concern of the Indonesian Air Force manpower planner.

Starting with the current out-of balanced force structure condition, this study is intended to develop some mathematical models to assist the manpower planner in establishing a reasonable promotion policy which can create and maintain a well-balanced force structure in the Indonesian Air Force.

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I. INTRODUCTION

A. PROBLEM MOTIVATION

A well-balanced manpower structure in a graded population as in the Indonesian Air Force personnel organization is absolutely necessary. The designing of proper promotion and recruitment policies is a vital part of ensuring good manpower structure.

Experience has shown that a promotion policy which looks reasonable in the beginning but in fact fails to maintain the balance of the structure, can easily introduce pressure on the organization to grow at the top. Having this situation is obviously undesirable for the organization, since it will arouse the problem of understaffed lower grades and increasing personnel budget expenditures.

Aware of this situation, the Indonesian Air Force like any other Armed Forces is still looking for a good manpower planning model, for its promotion and recruitment system. So far, the existing policy is based mostly upon individual judgements of the higher authority, and it involves considerable trial and error which varies according to the manpower situation at that time.

At the current time, the manpower structure in the Indonesian Air Force is slightly out of balance, i.e., the ratio of the officer's grade, NCO's grade and Enlisted grades is not the proper size, but is bigger in the top

grades. This condition indicates the weakness of previous policies. Furthermore, under the new Department of Defense manpower planning program, the current Air Force manpower structure needs some modification to fit the new armed forces manpower requirement.

Based on this circumstance, this study is intended to construct some mathematical models which might be useful to assist the Air Force manpower planner in achieving proper promotion and recruitment policies and control of force structure in the short run and long run basis.

B. DEFINITION OF THE PROBLEM

Given the current out of balance manpower structure in the Indonesian Air Force, find an optimal planning model which can create and maintain a well-balanced force structure, within some reasonable period, subject to constraints on promotion, recruitment, attrition and budget limitation.

C. SCOPE OF THE STUDY

In this study, the model considers the ranks (grades) structure only; separate categories for the military occupational specialty and individual length of service are not included in the study, due to data availability and a desire to simplify the model.

II. <u>PERSONNEL PROMOTION AND RECRUITMENT SYSTEM IN THE INDONESIAN AIR FORCE</u>

A. RANK STRUCTURE

In the Indonesian AirForce, the ranks are grouped into three major categories, i.e.;

a. Enlisted Personnel ("Tamtama")

b. NCO ("Bintara")

c. Officer ("Perwira")

- The enlisted personnel category consists of 4 ranks

1. 2nd private ("perajurit dua")
2. 1st private ("perajurit satu")
3. 2nd corporal ("koperal dua")
4. 1st corporal ("koperal satu")

- The NCO category consists of 7 ranks

1. 2nd sergeant ("sersan dua")
2. 1st sergeant ("sersan satu")
3. master sergeant ("sersan kepala")
4. sergeant major ("sersan major")
5. 2nd assistant officer (pembantu letnan dua")
6. 1st assistant officer ("pembantu letnan satu")
7. candidate officer ("calon perwira")

- The officer category consists of 10 ranks, which can also be classified into three more categories, i.e.,

a. Lower ranking officer

1. 2nd lieutenant ("letnan dua")
2. 1st lieutenant ("letnan satu")
3. captain ("kapten")

b. Middle ranking officer

1. Major ("mayor")

2. Lieutenant Colonel ("Letnan Kolonel")

3. Colonel ("Kolonel")

c. Higher ranking officer

1. 1st Air Marshal ("Marsekal pertama")

2. Air Vice Marshal ("Marsekal muda")

3. Air Chief Marshal ("Marsekal madya")

4. Air Marshal ("Marsekal")

According to the Department of Defense manpower program, the desired proportion of rank's composition of each Armed Force was stated in Table-1 [7]

TABLE-1 Group rank's ratio for every Armed Force

Organization	Officer	NCO	Enlisted
Army	1	5	9
Navy	1	4	5
Air Force	1	5	4
Police Force	1	6	5

Also, Table-2 shows the Air Force Officer's ratios as recommended by the Department of Defense.

TABLE-2 Air Force Officers Ratios

Grade	Max % relative to total A/F officers
High ranking officers	1%
Colonels	4%
Lieutenant Colonels	15%
Majors	20%
Captains	25%
Lieutenants	35%
TOTAL	100%

B. TRAINING REQUIREMENTS

Training has an important function in developing the quality of the personnel and the organization. Some promotions demand a certain type or level of training as one of their conditions. Therefore in developing personnel promotion strategies, it is necessary to put training into consideration, especially since in this study the training budget is one of the resources.

There are many types of training in the Air Force; however for the purpose of the study, only a few will be mentioned. These are the ones which have a direct effect on the promotion system. Table-3 shows the list of some of

TABLE-3 Types of Training/School in the Indonesian Air Force [3]

Types of Schools	Student Input	School's Duration
1. SEDASBATA	1st private+civilian with secondary school diploma	4 months
2. SECABA + DIKJURBA I	1st corporal	6 + 3 months
3. DIKJURBA II	Sergeant major	6 months
4. SECAPA + DIKJURBA III	1st assistant officer	5 + 6 months
5. DIKSARCAB	2nd lieutenant	6 months
6. SEKKAU	captain	6 months
7. AKABRI*	civilian with high school diploma	4 years
8. SEPAWAMIL**	civilian with college or university graduate	6 months

*Its training budget under special institution

**Its training budget under Department of Defense

of these training schools. It does not describe in detail the specialty of each school, i.e. aircraft maintenance, avionics, etc., since the models will not cover this degree of detail.

C. PROMOTION POLICIES AND ATTRITION

Basic Promotion Policies [7]

1. Rank advancement should be based on selectivity. The degree of selectiveness is increasing as the rank goes to the top, i.e. the higher the rank, the more difficult it is to be promoted.
2. The prime consideration for rank promotion is personnel conduct and performance, whereas other factors are just supporting factors.
3. Some minimal length of service in the previous rank is necessary to be fulfilled before promotion to upper ranks can occur.
4. The rate of people promoted each year, should be matched with the desired force structure of the corresponding year.
5. Some people who fail permanently to be promoted should be given opportunity to leave the organization, and in some cases forced retirement may be necessary.

Types of Promotion (Rank Advancement)

1. Effective promotion: This is a regular promotion, which is based on general promotion procedure.

2. Extraordinary promotion: Promotions which are not regular usually happen in war or equivalent operations, due to extraordinary performance such as heroism. Like a regular promotion, it also has full adminstrative effect.

3. Honorary promotion: Honorary promotion is given to people who have shown excellent performance during service, but due to some reasons cannot be given a regular promotion. This promotion is awarded right before retirement, and is valid for all ranks up to Vice Marshal (as long as they have fulfilled the retirement requirement).

4. Posthumous promotion: Posthumous promotion is given as a respect to people who have already died and who had shown outstanding performance during the war or great contribution to the organization.

For modelling purposes, the type of promotion which will be taken into account, is just the regular promotion system, since it has a definite rule and significant influence on the whole force structure development. The other types only create slight change in the system; hence disregarding these types will not cause major error in computation.

Table 4 and Table 5 show time limitations for the officers and NCO promotion system in the Armed Forces based on Department of Defense policies [7].

TABLE-4. Time Limitations for Officer's Promotion System

Rank's Advancement	Time Limitations
From 2nd lieutenant to 1st lieutenant	2 - 5 years as an officer
From 1st lieutenant to captain	5 - 9 years as an officer
From captain to major	9-13 years as an officer
From major to Lieutenant colonel	13-17 years as an officer
From lieutenant Colonel to colonel	17-23 years as an officer
From colonel to higher ranking officer	Undefined

TABLE-5. Time Limitations for NCO's Promotion System

Rank's Advancement	Time Limitation
2nd sergeant to 1st sergeant	3-5 years as 2nd sergeant
1st sergeant to master sergeant	3-5 years as 1st sergeant
master sergeant to sergeant major	2-5 years as master sergeant
sergeant major to 2nd assistant officer	5-10 years as sergeant major
2nd assistant officer to 1st assistant officer	3-5 years as 2nd assistant officer
1st assistant officer to candidate officer	undefined

For enlisted personnel, from 2nd private up to 1st corporal regular promotion occurs whenever minimal time requirements have been completed, except if the individual has a very bad conduct record. But for 1st corporal to 2nd sergeant (transition from Enlisted to NCO), as well as 1st assistant officer to candidate officer (transition from NCO to officer) the time limitations are hard to define, since there exist some other factors which are more important than time in rank (such as education background and attitude requirement for that kind of rank).

For better illustration Figure 1 shows the interrelationship between ranks and training requirements, which describe the whole promotion system in the Air Force.

The ranks of private and above lieutenant colonel rank are not presented on that figure, since they will not be covered in the discussion. One reason for leaving higher ranks out of the model is that promotion above lieutenant colonel is very difficult to predict or forecast, because it involves too many individual judgements and political considerations.

As shown in the figure, there are three new inputs coming to the system every year for refreshing and balancing the structure. Those three inputs are

1. Civilians (with secondary school diploma) and 1st private ranks who go to 2nd corporal rank via "SEDASBATA" school.

2. "AKABRI" graduate input who become active 2nd lieutenant rank via "DIKSARCAB" school.

3. Civilians (with college/university diploma) who become 1st lieutenant via "SEPAWAMIL" school.

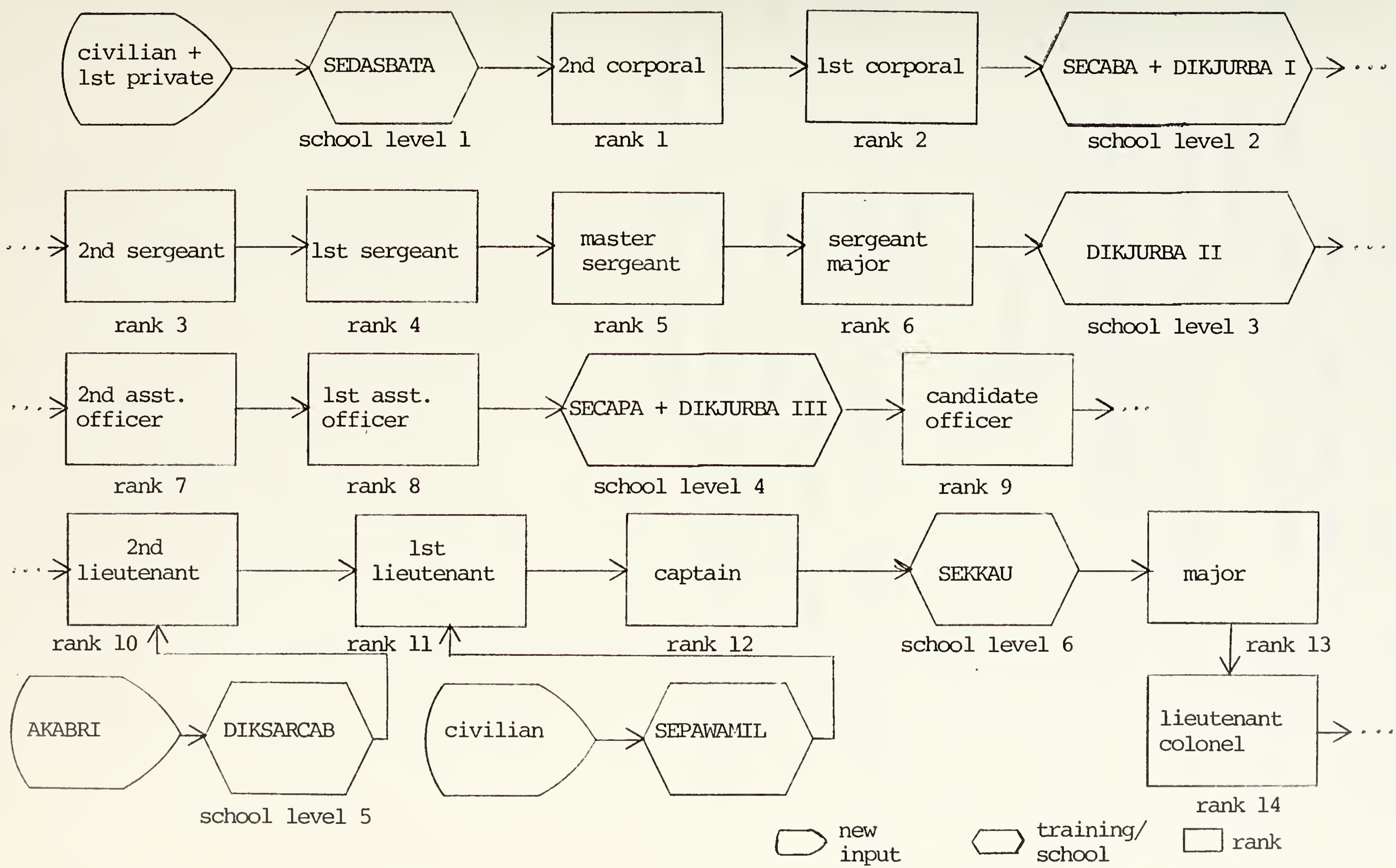


FIGURE 1. Air Force Personnel Promotion System

As well as these input flows, there is also an output flow due to attrition. Attrition might occur every year for all ranks, due to several different reasons such as:

1. Retirement
2. Contract ended (and not extended)
3. Physical and/or mental deficiency for performing regular activity
4. Reduction in Force
5. Voluntary resignation
6. Death
7. Other reasons.

Some of these types of attrition are beyond the control of the personnel planner, however based on past data and experiences, the controllable attritions can be used to influence the system and the uncontrollable attritions can be estimated fairly accurately.

III. MATHEMATICAL MODEL CONSTRUCTION

A. PROBLEM FORMULATION AND SOME UNDERLYING ASSUMPTIONS

The movement of personnel from grade to grade and movement of personnel from outside into the organization or vice versa, may be thought of as flow from state to state in the ordinary manpower model. The promotion and recruitment policy as well as attrition, can be treated as the flow parameters of the model, where the shape and the size of the force structures are highly dependent on them. Finding the right value of these parameters, i.e. the promotion, recruitment and attrition policy which can create a well-balanced force structure as required by the organization, is the main objective of the model.

Based on current situation of the Air Force manpower structure, the first goal of this study is to develop a model that can be used to move from the out-of-balance force structure to a well-balanced structure, within some definite and reasonable period. Once the well-balanced condition has been achieved, another model is needed to maintain the stability of that structure, either on a short run basis or a long run basis. According to the objectives of those models, then, the first model described above may be thought of as a "transition model", and the second model as a "steady-state model". For the purpose of the discussion, henceforth the first model will be called Model I. The

second model, i.e. the steady state model will be called Model II (for the long run basis), Model III (for the short run basis) and Model IV (for the short run cross-sectional model).

In developing all of these models certain major assumptions have been used, i.e.

1. There are enough people available to be recruited each year for enlisted rank input.

2. There exists a regular input to 2nd lieutenant rank, coming from Air Force Academy (AKABRI) graduates and also from senior NCO's each year.

3. There exists a new input to 1st lieutenant from college / university graduates each year.

4. a. Promotions exist in all ranks each year.

   b. Rank advancement can only happen one grade at a time

   c. There is no backward flow, i.e. movement from upper rank to lower rank in the system.

5. a. Attrition exists throughout the year in all ranks.

   b. Forced attrition is possible if it is necessary.

6. There are enough training facilities to support the personnel promotion requirement.

7. There exists a certain amount of cost attached to every personnel training and attrition.

8. There exists a limited budget allocated for personnel expenditure each year.

9. Since the retirement cost is under the different budget of another institution, it will not appear in this model.

10. The desired force structure has been determined in advance to agree with the job occupational requirement of the organization.

B. FORMULATION OF THE BASIC MODEL AND NOTATION

The Air Force manpower system in this study, can be described as a flow model as exhibited in Figure 2. The mathematical model which will be formulated later is basically derived from this flow model. Figure 2 shows this system in a flow diagram, where the stock (number) of people in each rank is represented in a box, the nodes show the level of training required for the corresponding promotions and the arrows indicate the directions of the personnel movements. As shown in the diagram, the manpower system evolves over time. New individuals join the system, and individuals in the system remain in one grade for a time, then either move to another grade or leave the system. At certain points in time $t = 0,1,2,...$ we imagine that all motion in the system stops and we count the number of individuals in each grade. By this assumption then we can observe and count the number (stock) of people in each grade, and compute the number of attritions and the number of promotions which have occurred since the previous time period. Before getting into the more detailed discussion of this model, to avoid unnecessary confusion, we introduce the notation which will be used for this model.

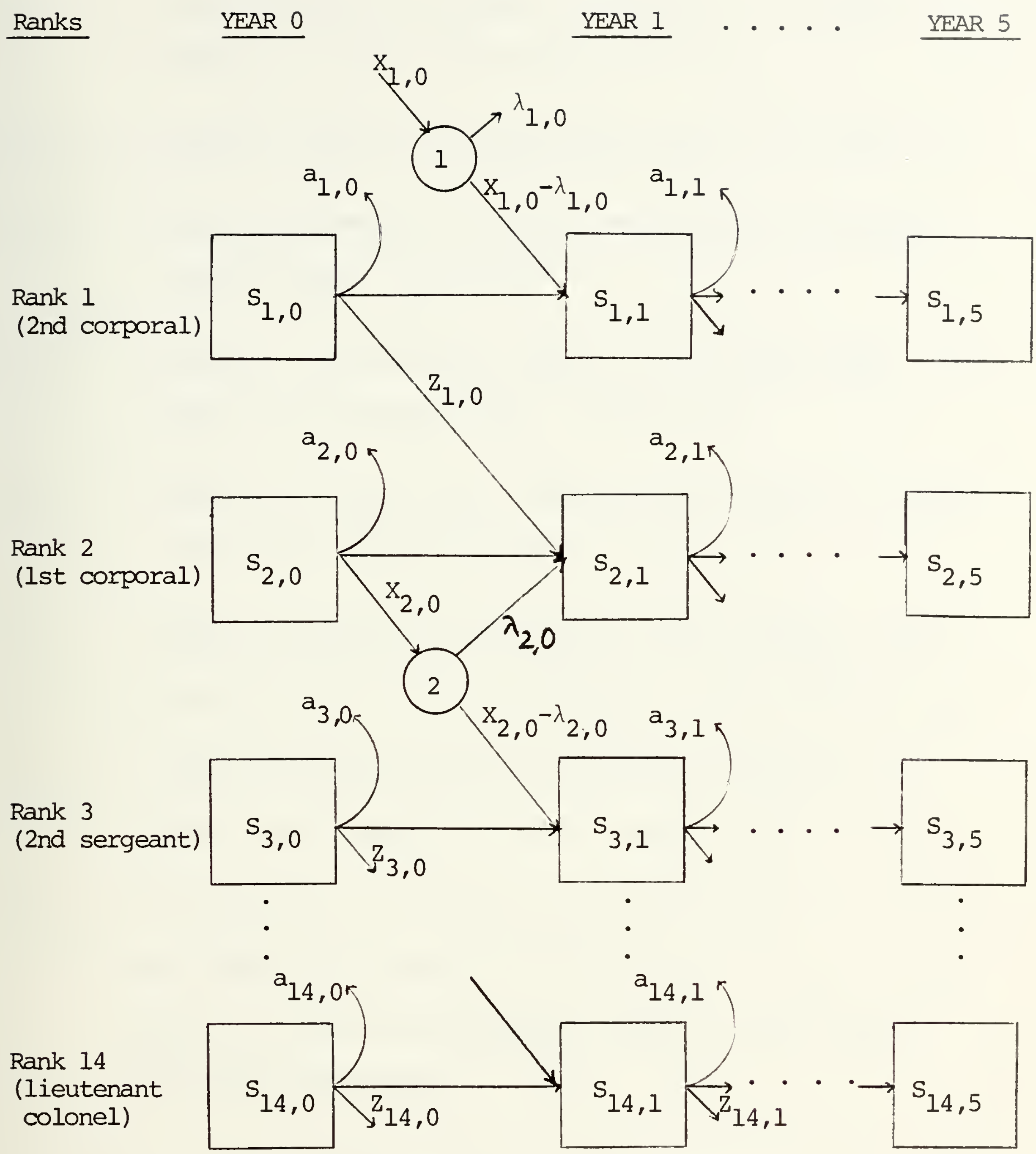


FIGURE 2. Flow Diagram of Personnel Movement

$S_{i,t}$ = number (stock) of people with rank i at time t, where $i = 1,2,\ldots,14$ and $t = 0,1,2,\ldots$

$S_{i,t+1}$ = number (stock) of people with rank i at time t+1

$a_{i,t}$ = number of people of rank i who leave the system during time t to time t+1

$Z_{i,t}$ = number of people who were promoted from rank i to rank i+1, during time t to time t+1

$X_{j,t}$ = number of people who attend school level j during time t to time t+1 where $j = 1,2,\ldots,6$

$\lambda_{j,t}$ = number of people who drop out from school level j, during time t to t+1 and back to the previous rank

$Y_{i,t}$ = number of people from university graduate who enter the system as rank i, during time t to t+1

Note: if there exists a training requirement for promotion from rank i to rank i+1, then $Z_{i,t} = X_{j,t} - \lambda_{j,t}$, where j is the corresponding school level needed for that requirement

In this case, some of the Z_i's can be represented as X_i's such as: Refer to Figure 1 on page 23):

$$Z_{2,t} = X_{2,t} - \lambda_{2,t}$$

$$Z_{6,t} = X_{3,t} - \lambda_{3,t}$$

$$Z_{8,t} = X_{4,t} - \lambda_{4,t}$$

$$Z_{12,t} = X_{6,t} - \lambda_{6,t}$$

While for the new input we have:

$X_{1,t} - \lambda_{1,t}$ = new input of rank 1

$X_{5,t} - \lambda_{5,t}$ = new input of rank 10

$Y_{11,t}$ = new input of rank 11.

Based on the flow diagram in Figure 2, we can construct some mathematical models for this system with the following equations, which give the force structure at time t+1 in terms of the above defined variables.

$$S_{1,t+1} = S_{1,t} - a_{1,t} - Z_{1,t} + X_{1,t} - \lambda_{1,t} \tag{1}$$

$$S_{2,t+1} = S_{2,t} - a_{2,t} + Z_{1,t} - X_{2,t} + \lambda_{2,t} \tag{2}$$

$$S_{3,t+1} = S_{3,t} - a_{3,t} - Z_{3,t} + X_{2,t} - \lambda_{2,t} \tag{3}$$

$$S_{4,t+1} = S_{4,t} - a_{4,t} + Z_{3,t} - Z_{4,t} \tag{4}$$

$$S_{5,t+1} = S_{5,t} - a_{5,t} + Z_{4,t} - Z_{5,t} \tag{5}$$

$$S_{6,t+1} = S_{6,t} - a_{6,t} + Z_{5,t} - X_{3,t} + \lambda_{3,t} \quad (6)$$

$$S_{7,t+1} = S_{7,t} - a_{7,t} - Z_{7,t} + X_{3,t} - \lambda_{3,t} \quad (7)$$

$$S_{8,t+1} = S_{8,t} - a_{8,t} + Z_{7,t} - X_{4,t} + \lambda_{4,t} \quad (8)$$

$$S_{9,t+1} = S_{9,t} - a_{9,t} - Z_{9,t} + X_{4,t} - \lambda_{4,t} \quad (9)$$

$$S_{10,t+1} = S_{10,t} - a_{10,t} + Z_{9,t} - Z_{10,t} + X_{5,t} - \lambda_{5,t} \quad (10)$$

$$S_{11,t+1} = S_{11,t} - a_{11,t} + Y_{11,t} + Z_{10,t} - Z_{11,t} \quad (11)$$

$$S_{12,t+1} = S_{12,t} - a_{12,t} + Z_{11,t} - X_{6,t} + \lambda_{6,t} \quad (12)$$

$$S_{13,t+1} = S_{13,t} - a_{13,t} - Z_{13,t} + X_{6,t} - \lambda_{6,t} \quad (13)$$

$$S_{14,t+1} = S_{14,t} - a_{14,t} + Z_{13,t} - Z_{14,t} \quad (14)$$

Notice in this model there are only 14 different classes (ranks or grades) considered, i.e. $S_i = \{S_i; i=1,2,...,14\}$ 6 levels of training, i.e. $X_i = \{X_i; i=1,2,...,6\}$ and 3 new inputs, i.e. $(X_i - \lambda_1)$ as rank 1's input; $(X_5 - \lambda_5)$ as rank 10's input; and Y_{11} as rank 11's input. Those 14 equations which have just been developed will become the basic flow model of the manpower models which will be constructed later in this study.

C. MODEL 1 (RESHAPING THE PROPER RATIO BETWEEN RANKS)

As has been mentioned in section III.a, the first requirement for solving the current Air Force manpower problem is to establish a well-balanced force structure within some reasonable period (we take 5 years in this case). Hence we can define the problem for the first model as follows:

Given the current out-of-balance force structure, find proper promotion, recruitment and attrition policies which yield a well-balanced force structure within a 5 year planning program, and satisfying these following conditions:

(1) Total number of people in each year should not be less than the minimum manpower requirement.

(2) Total number of people in each year should not be more than can be afforded by the available budget.

(3) Total number of people in the system at year 5, should be equal or at least close to the number recommended by the Department of Defense.

(4) The ratios between officers, NCO's and Enlisted men should agree with the desired Air Force organization policy.

(5) The number of people being promoted and the attrition should not be drastically changed, i.e. within reasonable number which is not far from the latest policy.

To get an optimum well-balanced force structure, i.e. a well-structured organization with minimum number of personnel, and still satisfying those conditions, a linear programming method will be applied to this problem. We let the objective function be to minimize the total number of personnel, subject to

a. flow constraints

b. policy constraints

c. stock constraints

d. budget constraint.

And let the promotion, recruitment and attrition policies be the decision variables; then Model I will be constructed as follows:

Objective Function

Since the main goal of this model is to get an optimum well-balanced force structure within 5 year planning program then we can construct the objective function as:

$$\text{Min} \sum_{i=1}^{14} S_{i,t} \qquad t = 5$$

Subject to:
- Flow constraints
- Policy Constraints
- Stock Constraints
- Budget Constraints

$\sum_{i=1}^{14} S_{i,t}$ = total number of people in the system at year t

(1) Flow Contraints

The whole basic flow model which consists of 14 equations as shown on pages 25 and 26, is used as the flow constraints of this model, where $t = 0,1,2,\ldots,4$.

(2) Policy Constraints

The number of people to be promoted, recruited or leaving the system each year, all should be within reasonable limits that is feasible with the real world situation.

Let $S_{i,t}$ = number (stock) of people with rank i at time t

α_i = maximum fraction of people being promoted each year relative to $S_{i,t}$ where

$0 \leq \alpha_i < 1 \qquad i = 1,2,\ldots,14 \qquad t = 0,1,\ldots,4$

β_i = minimum fraction of people being promoted each year relative to $S_{i,t}$, where $0 \leq \beta_i < \alpha_i$

n_j = minimum number of people entering school level j each year, $j = 1,5$

m_j = maximum number of people entering school level j each year, $j = 1,5$

k_i = minimum number of people entering rank i each year

o_i = maximum number of people entering rank i each year

μ_i = maximum fraction of people leaving the system from rank i each year relative to $S_{i,t}$

$0 \leq \mu_i < 1 \qquad i = 1,2,\ldots,14$

$\lambda_{j,t}$ = number of people/student who drop out from school level j at year t

γ_j = maximum fraction of people/student who drop out from school level j each year relative to $X_{j,t}$

$X_{j,t}$ = number of people/student attending school level j during period time t to t+1, j = 1,2,...,6

$Z_{i,t}$ = number of people being promoted from rank i to rank i+1 during period time to to t+1.

Notes for i = 2,6,8,12

$$Z_{2,t} = X_{2,t} - \lambda_{2,t}$$

$$Z_{6,t} = X_{3,t} - \lambda_{3,t}$$

$$Z_{8,t} = X_{4,t} - \lambda_{4,t}$$

$$Z_{12,t} = X_{6,t} - \lambda_{6,t}$$

Then the policy constraints will be:

$$\beta_i S_{i,t} \leq Z_{i,t} \leq \alpha_i S_{i,t} \qquad i;\ t = 0,1,\ldots,4 \qquad (15)$$

$$n_j \leq X_{j,t} \leq m_j \qquad j = 1,5;\ t = 0,1,\ldots,4 \qquad (16)$$

$$k_i \leq Y_{i,t} \leq o_i \qquad i = 11;\ t = 0,1,\ldots,4 \qquad (17)$$

$$0 \leq a_{i,t} \leq \mu_i S_{i,t} \qquad i;\ t = 0,1,\ldots,4 \qquad (18)$$

$$0 \leq \lambda_{j,t} \leq \gamma_j X_{j,t} \qquad j;\ t = 0,1,\ldots,4 \qquad (19)$$

All β_i's; α_i's, n_j's; m_j's, k_i's, o_i's, μ_i's and γ_j's are known coefficients, which are given by the manpower planner based from past data or reasonable justification.

(3) <u>Stock Constraints</u>

There are 7 types of constraints which should be considered in the stock constraints

(i) Total stock constraints

(ii) Total Enlisted's (Corporal rank) stock constraints

(iii) Total NCO's stock constraint

(iv) Total officer's stock constraint

(v) Corporal's rank proportion constraint

(vi) NCO's rank proportion constraints

(vii) Officer's rank proportion constraints

(a) Total stock constraint

There is a restriction that every year the total number of perople in the system should not be less than the minimum manpower requirement

$$\sum_{i=1}^{14} S_{i,t} \geq C \qquad i;\ t = 1,2,\ldots,5 \tag{20}$$

(b) Total Corporal's stock constraint

$$P_{1b} \sum_{i=1}^{14} S_{i,t} \leq \sum_{i=1}^{2} S_{i,t} \leq P_{1a} \sum_{i=1}^{14} S_{i,t} \qquad t=5 \tag{21}$$

$\sum_{i=1}^{14} S_{i,t}$ = total number of people in the system at year t

$\sum_{i=1}^{2} S_{i,t}$ = total number of people with rank 1 and rank 2 at year t

p_{1a} = maximum fraction of people with rank 1 and rank 2 with respect to the whole population of the same year $0 < p_{1a} < 1$

p_{1b} = minimum fraction of people with rank 1 and rank 2 with respect to the whole population of the same year $0 < p_{1b} < p_{1a} < 1$

Notes: For this model, i.e. the Transition Model or Model I, except for the total stock constraint in equation (20), the rest of the stock constraints will be used only for year 5, the reason is, to give more flexibility to the manpower planner in adjusting the out-of-balance situation to the well-balanced structure, during this transition period.

(c) Total NCO's stock constraint

$$p_{3b} \sum_{i=1}^{14} S_{i,t} \le \sum_{i=3}^{9} S_{i,t} \le p_{3a} \sum_{i=1}^{14} S_{i,t} \qquad t=5 \tag{22}$$

$\sum_{i=3}^{9} S_{i,t}$ = total number of people with NCO ranks at year t

$p_{3,\ldots}$ = fraction number of people with NCO's rank with respect to the total population.
index a indicates the upper bound
index b indicates the lower bound

$0 < p_{3b} < p_{3a} < 1$

(d) Total officers stock constraint

$$P_{10b} \sum_{i=1}^{14} S_{i,t} \le \sum_{i=10}^{14} S_{i,t} \le P_{10a} \sum_{i=1}^{14} S_{i,t} \quad t=5 \tag{23}$$

$\sum_{i=10}^{14} S_{i,t}$ = total number of people with officer's ranks at year t

$P_{10,..}$ = fraction number of people with officer ranks with respect to the total population

index a indicates the upper bound

index b indicates the lower bound

$P_{10b} < P_{10a} < 1$

(e) Corporal's rank proportion constraint

Besides the total stock proportion constraints as described in equations (21), (22) and (23), there must exist a proper ratio among the ranks within each group. For the Corporal's rank this constraint is

$$P_{1,0} \sum_{i=1}^{2} S_{i,t} \le S_{1,5} \le P_{1,1} \sum_{i=1}^{2} S_{i,t} \quad t=5 \tag{24}$$

$\sum_{i=1}^{2} S_{i,t}$ = total number of people with rank 1 and rank 2 at year t

$S_{1,t}$ = number of people with rank 1 at year t

$p_{1,0}$ = minimum fraction of people with rank 1 with respect to $\sum_{i=1}^{2} S_{i,t}$

$p_{1,1}$ = maximum fraction of people with rank 1 with respect to $\sum_{i=1}^{2} S_{i,t}$

(f) NCO's rank proportion constraints

Each NCO's rank from rank 3 to rank 9, should have a certain proportion with respect to the whole NCO's ranks. The lower the rank the bigger the proportion should be

$$p_{3,0} \sum_{i=3}^{9} S_{i,t} \leq S_{3,t} \leq p_{3,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{25}$$

$$p_{4,0} \sum_{i=3}^{9} S_{i,t} \leq S_{4,t} \leq p_{4,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{26}$$

$$p_{5,0} \sum_{i=3}^{9} S_{i,t} \leq S_{5,t} \leq p_{5,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{27}$$

$$p_{6,0} \sum_{i=3}^{9} S_{i,t} \leq S_{6,t} \leq p_{6,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{28}$$

$$p_{7,0} \sum_{i=3}^{9} S_{i,t} \leq S_{7,t} \leq p_{7,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{29}$$

$$p_{8,0} \sum_{i=3}^{9} S_{i,t} \leq S_{8,t} \leq p_{8,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{30}$$

$$p_{9,0} \sum_{i=3}^{9} S_{i,t} \leq S_{9,t} \leq p_{9,1} \sum_{i=3}^{9} S_{i,t} \qquad t=5 \tag{31}$$

$S_{i,t}$ = number of people with rank i at year t

$i = 3,4,\ldots,9$

$\sum_{i=3}^{9} S_{i,t}$ = total number of people with rank NCO at year t

$p_{i,0}$ = minimum fraction of people with rank i with respect to $\sum_{i=3}^{9} S_{i,t}$; $i = 3,4,\ldots,9$

$p_{i,1}$ = maximum fraction of people with rank i with respect to $\sum_{i=3}^{9} S_{i,t}$; $i = 3,4,\ldots,9$

where

$$p_{i,1} > p_{i+1,1} \qquad i = 3,4,\ldots,8$$

$$p_{i,0} > p_{i+1,0} \qquad i = 3,4,\ldots,8$$

$$p_{i,1} > p_{i,0} \qquad i = 3,4,\ldots,9$$

(g) Officer's rank proportion constraints

Similar to the situation in the NCO's rank proportion constraints, the officer's rank group also needs those requirements (recall Section II at table 2 on page 11).

$$S_{10,t} \leq p_{10,0} \sum_{i=10}^{11} S_{i,t} \qquad t=5 \tag{32}$$

$$p_{11,0} \sum_{i=10}^{14} S_{i,t} \leq \sum_{i=10}^{11} S_{i,t} \leq p_{11,1} \sum_{i=10}^{14} S_{i,t} \qquad t=5 \qquad (33)$$

$$p_{12,0} \sum_{i=10}^{14} S_{i,t} \leq S_{12,t} \leq p_{12,1} \sum_{i=10}^{14} S_{i,t} \qquad t=5 \qquad (34)$$

$$p_{13,0} \sum_{i=10}^{14} S_{i,t} \leq S_{13,t} \leq p_{13,1} \sum_{i=10}^{14} S_{i,t} \qquad t=5 \qquad (35)$$

$$p_{14,0} \sum_{i=10}^{14} S_{i,t} \leq S_{14,t} \leq p_{14,1} \sum_{i=10}^{14} S_{i,t} \qquad t=5 \qquad (36)$$

$p_{i,1}$ = maximum fraction of people with rank i (including p_{10}) with respect to total officers $\sum_{i=10}^{14} S_{i,t}$

$p_{i,0}$ = minimum fraction of people with rank i with respect to $\sum_{i=10}^{14} S_{i,t}$

where:

$p_{i,1} > p_{i+1,1}$ $\qquad i = 11,12,13$

$p_{i,0} > p_{i+1,1}$ $\qquad i = 11,12,13$

$p_{i,1} > p_{i,0}$ $\qquad i = 11,12,13.$

<u>Notes</u>: All p_i's in equations (21) up to equation (36) are assumed known coefficients or given by manpower planner.

(4) Budget Constraint

To keep the total personnel expenditure within a reasonable cost which does not exceed the allocated budget, the budget constraint should be considered for this model. The budget allocated for this expenditure actually consists of several different categories, however for the purpose of this study, there are only three major personnel cost categories introduces. These three personnel expenditures are: salary, training cost and attrition cost of each individual with rank i = (i = 1,2,...,14) per year.

Let

C_i = average salary of a people with rank i per year

e_j = average cost of training a people in school level j per period

g_i = average attrition cost of a people with rank i at the corresponding year

$X_{i,t}$ = number of people who attend school j during time t to t+1

$S_{i,t+1}$ = number of people with rank i at time t+1

$a_{i,t}$ = number of attrition during t to t+1

B_t = amount of budget available for time t to t+1

$$\sum_{i=1}^{14} C_i S_{i,t+1} + \sum_{i=1}^{14} g_i a_{i,t} + \sum_{j=1}^{6} e_j X_{j,t} \leq B_t \qquad (37)$$

$$t = 1,\ldots,4$$

D. MODEL II (MAINTAINING THE PROPER RATIO BETWEEN RANKS — LONG RUN MODEL)

The purpose of Model II is to give an aid to the manpower planner in maintaining a force structure which is already well-balanced through Model I, for another future period. In fact there are three models which can be used for this purpose. The differences among these models are:

a. Model II uses linear programming method based on 5 year planning program (long run)

b. Model III uses linear programming method based on 1 year planning program (short run)

c. Model IV uses cross-sectional method based on 1 year planning program (short run).

Model II will be discussed in this section; the other two models will be discussed later in another section of this paper.

Model II uses the same linear programming method as used in Model I in the previous section. The model is

$$\text{Min} \sum_{i=1}^{14} S_{i,t} \qquad t = 5$$

subject to:
- Flow constraints
- Policy constraints
- Stock constraints
- Budget Constraints

The objective function, the flow constraints, the policy constraints and the budget constraints used in Model II are exactly the same as in Model I, but the stock constraints are slightly different. There are more restrictions given to the stock constraints than in the previous model, however since the force structure is already well-balanced, the more restrictive constraints do not cause trouble.

The stock constraints in Model II, then will be

a. <u>Total Stock Constraint</u>

Exactly the same as equation (20) on page 31.

b. <u>Total corporal's stock constraint</u>

Similar to equation (21) on page 31, except now $t = 1,2,...,5$ instead of $t = 5$.

c. <u>Total NCO's Stock Constraint</u>

Similar to equation (22) on page 32, except now $t = 1,2,...,5$ instead of $t = 5$.

d. <u>Total Officer's Stock Constraint</u>

Similar to equation (23) on page 33 with $t = 1,2,...,5$.

e. <u>Corporal's Rank Proportion Constraint</u>

The Corporal rank proportion constraint causes problems in the steady state model. If we use this constraint in the steady state period, we can maintain the proportion of rank 1 and rank 2 in the proper condition as desired for a well-balanced force structure, but the consequence of using this constraint is that the size of the force structure becomes very large, since the system needs more and more input for rank 1 in order to maintain that proportion.

Eventually this makes the solution become infeasible when the budget is exhausted. IF we do not use this constraint, we are unable to maintain the desired proportion of rank 1 and rank 2, however we can keep the size of the force structure within reasonable numbers. Thus there exists a trade off situation concerning the applicaton of this constraint. For the sake of maintaining a minimum well-balanced force structure, we prefer to leave this constraint out of the model. For further explanation see the example of model II application on page 71.

f. <u>NCO's Rank Proportion Constraints</u>

Similar to equations (25), (26), (27), (28), (29), (30) and (31) on page 34, except t = 1,2,...,5.

g. <u>Officer's Rank Proportion Constraints</u>

Similar to equations (33), (34), (35) and (36) on page 36, except t = 1,2,...,5.

Notice equation (32) is also not being used in this model, based on the same problem which exists with the corporal's rank proportion constraints.

E. MODEL III (MAINTAINING THE PROPER RATIO BETWEEN RANKS — SHORT RUN MODEL)

As in Model II, Model III is also a kind of steady state model. Although both of these models have the same objective, the planning period used on their approach is different. Model III used a one year planning period instead of a 5 year planning program. However, this model is also applicable for

a 5 year planning program if necessary, by just running the model 5 times repeatedly. The linear programming model used for this model is similar to Model II, except the planning period is 1 year instead of 5 years. Hence by changing the timing convention used in Model II we get Model III. Model III is as follows:

$$\text{Minimize} \quad \sum_{i=1}^{14} S_{i,t} \qquad t = 1$$

Subject to: - Flow constraints

- Policy constraints

- Stock constraints

- Budget constraint

In order to give a better illustration about the difference among Model I, Model II and Model III, Table 4 has been set up for this purpose.

F. MODEL IV (MAINTAINING THE PROPER RATIO BETWEEN RANKS — CROSS SECTIONAL MODEL)

This model basically has the same objective as Model III i.e., maintaining the well-balanced force structure, but the approach is completely different. In Model III the promotion, recruitment and attrition policies are the decision variables which are not known prior to the computation, whereas in Model IV all of these variables are already known, i.e. the manpower planner has already defined what kind of policies are going to be used for the next planning program.

TABLE 4. The Difference Between Model I, Model II and Model III

	Model I	Model II	Model III
A. Objective function	t=5	t=5	t=1
B. Flow constraints	t=0,1,...,4	t=0,1,...,4	t=0
C. Policy constraints	t=0,1,...,4	t=0,1,...,4	t=0
D. Stock constraints			
1. Total stock constraint	t=1,2,...,5	t=1,2,...,5	t=1
2. Total corporal's stock constraint	t=5	t=1,2,...,5	t=1
3. Total NCO's stock constraint	t=5	t=1,2,...,5	t=1
4. Total officer's stock constraint	t=5	t=1,2,...,5	t=1
5. Corporal's rank proportion constraint	t=5	None	None
6. NCO's rank proportion constraint	t=5	t=1,2,...,5	t=1
7. Officer's rank proportion constraints	t=5	t=1,2,...,5 [lieutenant constraint omitted]	t=1 [lieutenant constraint omitted]
E. Budget constraints	t=0,1,...,4	t=0,1,...,4	t=0,1

Consequently the objective is just to predict the size of the future force structure, based on the planned policies. In order to use Model IV, the manpower planner must first decide which kinds of policies (flow parameters) to use in order to obtain the desired force structure.

The solutions yielded from Model II and Model III might be appropriate for estimating these flow parameters. Once these flow parameters have been defined, a flow matrix can be constructed for this model.

Let

$f_{i,j}(t)$ = number of people moving from rank i to rank j during time t to t+1

If i = 0 means a movement into the system

If j = 0 means a movement out of the system

If i = j means staying at that rank.

$S_i(t)$ = number of people with rank i at time t

$q_{j,i}$ = fraction of people moving from rank i to rank j during time t to t+1. (Notice the indexes are the opposite of $f_{i,j}$) and q is independent of time)

The movement of the people in the system can be described as in the flow diagram in Figure 3.

$$q_{j,i} = \frac{f_{i,j}(t)}{S_i(t)} \tag{38}$$

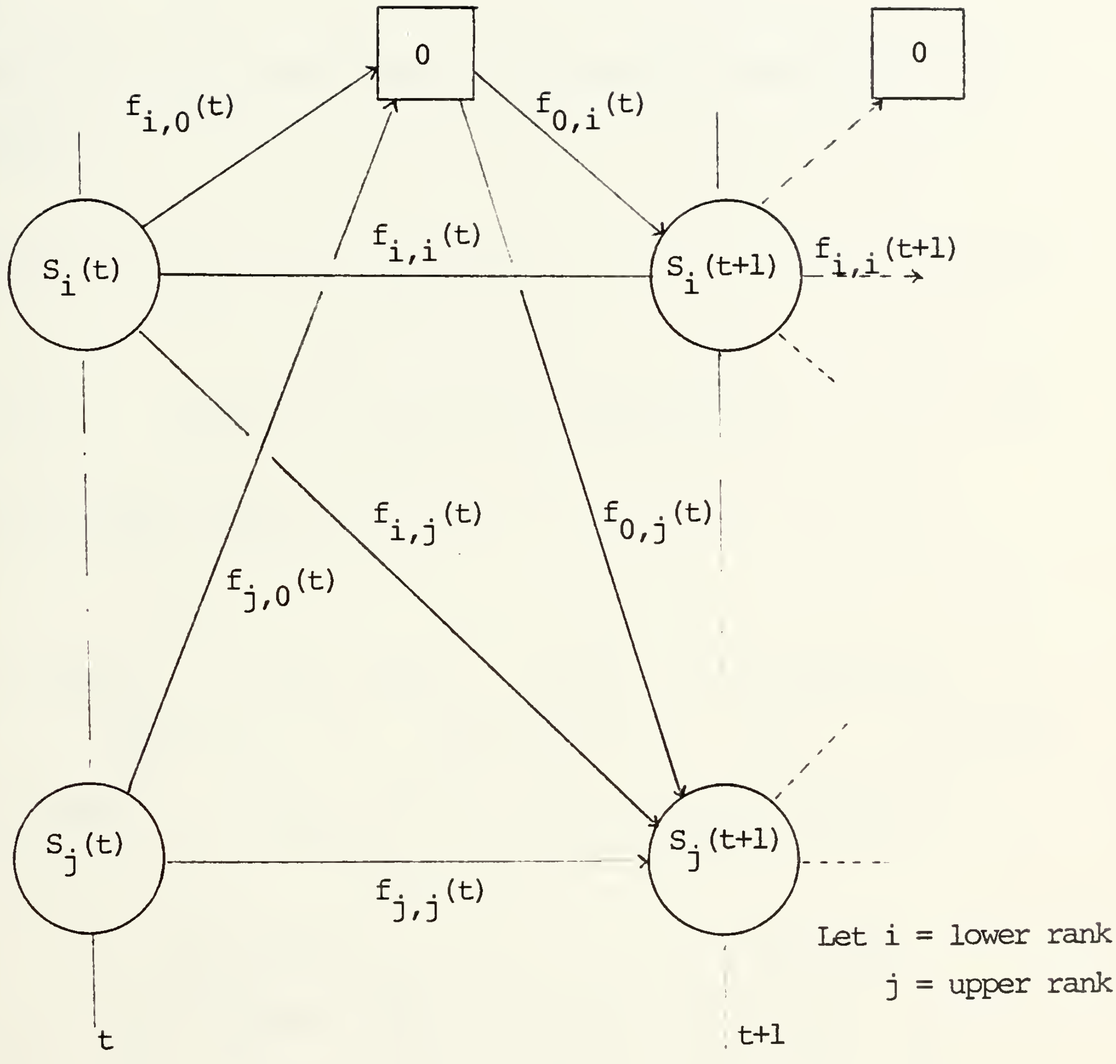


FIGURE 3. Cross-Sectional Model

From Figure 3 we can derive the equation for the cross-sectional model [1]:

$$S_i(t+1) = f_{o,i}(t) + f_{i,i}(t) \tag{39}$$

$$S_j(t+1) = f_{o,j}(t) + f_{i,j}(t) + f_{j,j}(t) \tag{40}$$

From equation (38), these last two equations can be written as follows:

$$S_i(t+1) = f_{o,i}(t) + q_{i,i}S_i(t) \tag{41}$$

$$S_j(t+1) = f_{o,j}(t) + q_{j,i}S_i(t) + q_{j,j}S_j(t) \tag{42}$$

or in matrix form will be:

$$\begin{pmatrix} S_i(t+1) \\ \\ S_j(t+1) \end{pmatrix} = \begin{pmatrix} f_{o,i}(t) \\ \\ f_{o,j}(t) \end{pmatrix} + \begin{pmatrix} q_{i,i} & 0 \\ \\ q_{j,i} & q_{j,j} \end{pmatrix} \begin{pmatrix} S_i(t) \\ \\ S_j(t) \end{pmatrix} \tag{43}$$

Let $S(t+1) = \begin{pmatrix} S_1(t+1) \\ S_2(t+1) \\ \vdots \\ S_N(t+1) \end{pmatrix}$; $f_0(t) = \begin{pmatrix} f_{0,1}(t) \\ f_{0,2}(t) \\ \vdots \\ f_{0,N}(t) \end{pmatrix}$

$$Q = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1N} \\ q_{21} & q_{22} & \cdots & q_{2N} \\ \vdots & \vdots & & \vdots \\ q_{N1} & q_{N2} & \cdots & q_{NN} \end{pmatrix}$$

$$S(t) = \begin{pmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_N(t) \end{pmatrix}$$

Then equation (43) will become

$$S(t+1) = f_0(t) + Q\,S(t) \tag{44}$$

$q_{i,i}$ can be obtained from $f_{i,i}$ as follows,

$$f_{i,i}(t) = S_i(t) - f_{i,0}(t) - f_{i,j}(t)$$

$$= S_i(t) - q_{0,i}S_i(t) - q_{j,i}S_i(t)$$

since $f_{i,i}(t) = q_{i,i}S_i(t)$.

Then, $$q_{ii}S_i(t) = S_i(t) - q_{0,i}S_i(t) - q_{j,i}S_i(t)$$

$$q_{ii} = 1 - q_{0,i} - q_{j,i} \tag{45}$$

Notice in this case the flow matrix Q does not include state 0 in its structure, hence the sum of its columns does not equal 1.

For this Air Force manpower model, the size of flow matrix Q will be 14 x 14, the new appointment policy vector $f_0(t)$ will be 14 x 1, and the stock size vector, $S_i(t+1)$ and $S_i(t)$ will be 14 x 1. Specifically, Model IV can be described as in equation (46):

$$\underset{14\times1}{\begin{bmatrix} S_1(t+1) \\ S_2(t+1) \\ \vdots \\ \\ \\ \\ \\ \\ S_{14}(t+1) \end{bmatrix}} = \underset{14\times1}{\begin{bmatrix} f_{0,1}(t) \\ 0 \\ 0 \\ \vdots \\ f_{0,10}(t) \\ f_{0,11}(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}} + \underset{14\times14}{\begin{bmatrix} q_{11} & 0 & 0 & \cdots & & & \\ q_{21} & q_{22} & 0 & \cdots & & & \\ 0 & q_{32} & q_{33} & 0 & \cdots & & \\ 0 & 0 & q_{43} & q_{44} & & & \\ 0 & 0 & & & & & \\ \vdots & & & & & & \\ & & & & & & 0 \\ 0 & & & & 0 & q_{14,13} & q_{14,14} \end{bmatrix}} \underset{14\times1}{\begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ \\ \\ \\ S_{14}(t) \end{bmatrix}} \tag{46}$$

Notes: 1. There are 14 types of rank $S_i(t) = \{S_i(t);\ i=1,2,\ldots,14\}$

2. There are only 3 new inputs each year $f_{0,1}(t)$; $f_{0,10}(t)$ and $f_{0,11}(t)$.

3. There is no movement from upper ranks to lower ranks, i.e. no $q_{j,i}$ with $i > j$

4. There is only one step promotion or stay in the same rank each year

   i.e., $q_{j,i}$ should have $j = i$ or $j = i+1$.

Knowing the promotion and attrition policies (flow matrix Q), the recruitment policies, i.e. the new appointment policy ($f_0(t)$) and the current force structure ($S_i(t)$), the manpower planner can use this model to predict the future force structure. If it is shown that the resulting future force structure will be worse or undesirable, then by slightly adjusting the value of its Q matrix, and $f_0(t)$, they may be able to improve the predicted structure.

IV. APPLICATION OF THE MODELS

The purpose of this section is to exhibit some numerical examples of the implications of the models discussed in the previous chapter. Through these various examples, we can compare the advantages and the disadvantages of each of the models as compared with the others. All of these examples, are taken from arbitrary data, which has been created by the author to be a reasonable approximation to the real world situation in the Indonesian Air Force. A separate example will be given for each model. In order to show the relation between the models and to make the comparison of the models easier, the problems used for each example are all based on the same data.

A. DATA RESOURCES AND PREPARATION

Before starting to demonstrate the numerical examples of the application of the models, it is worthwhile to discuss how the data are collected or retrieved from the available resources. This kind of activity is necessary to perform correctly in approaching the problem, otherwise bad data will be obtained and consequently will yield a wrong solution.

Assuming we already know what the Air Force manpower goal is (as has been mentioned in the previous sections), then what we need for the models is the following information:

(1) The existing force structre, i.e. the current stock level $\{S_{i,0};\ i=1,2,...,14\}$

(2) The average fraction of people being promoted each year in each rank $[Z_{i,t}/S_{i,t}]$

(3) The average fraction of people leaving the Air Force each year in every rank $[a_{i,t}/S_{i,t}]$

(4) Maximum number of people/student which can be accomodated or afforded in each school each year $(X_{j,t}; j = 1.6)$

(5) Average fraction of students who drop out from each school, each year $(\lambda_{j,t}/X_{j,t})$

(6) Proportion among ranks which will be achieved by the Air Force as required by the Department of Defense manpower planning program $[p_i$'s]

(7) Minimum number of Air Force personnel which should be maintained throughout the year, in order to prevent a shortage of manpower [C]

(8) Average cost of each personnel in certain rank each year $[c_i]$

(9) Average cost of training per personnel each year in certain school $[e_j]$

(10) Average attrition cost for each personnel in certain rank $[g_i]$

(11) Budget available for personnel expenditures each year $[B_t]$

(12) Average number of people entering the Air Force each year $[X_{1,t}$, $X_{5,t}$ and $Y_{11,t}]$

All of this information can be obtained through extensive data gathering from several different departments, i.e. Personnel department, Training department and Planning Programming Budgeting department and also some other departments concerned with this information.

In this case for example we can get the data with these following observations:

(1) The current stock level $[S_{i,0}]$ can be obtained from the personnel department, which contains the information of number of people in each rank at that year.

(2) The average fraction of people being promoted each year can be obtained from the past promotion record as it is exhibited in Table 5

TABLE 5 Promotion Record

	1970 - 1975			
Rank	1970	1971	...	1975
Rank 1 – Rank 2	$Z_{1,0}$	$Z_{1,1}$	. . .	$Z_{1,5}$
Rank 2 – Rank 5	$Z_{2,0}$	$Z_{2,1}$	. . .	$Z_{2,5}$
⋮	⋮	⋮		⋮
Rank 14 – Rank 5	$Z_{14,0}$	$Z_{14,1}$	. . .	$Z_{14,5}$

Then the average fraction of people promoted from rank i to rank i+1 is

$$\beta_i = \frac{1}{T} \sum_{t=0}^{T} \left(\frac{Z_{i,t}}{S_{i,t}} \right)$$

or β_i can be created by the manpower planner, if a new policy is to be applied.

Another method which is also quite reasonable to use is based on the average lifetime in any rank when the system is in equilibrium [1].

Let

s_i = number of people with rank i

f_{ii} = number of people who stay at rank i

$s_i - f_{ii}$ = number of new arrivals in rank i at each period

$l_{i,k}$ = the lifetime in rank i of arrival k where

where $k = 1,2,\ldots,s_i - f_{ii}$

Then the average lifetime l_i is

$$l_i = \frac{1}{s_i - f_{ii}} \sum_{k=i}^{s_i - f_{ii}} l_{i,k}$$

For m = 1,2,... and n_m be the number of arrivals with lifetime equal m (number of people who remain on rank i in m period). It follows that:

$$s_i - f_{ii} = \sum_{m=1}^{\infty} n_m$$

and

$$l_i = \frac{1}{s_i - f_{ii}} \sum_{m=1}^{\infty} mn_m$$

Under the assumption of equilibrium then $l_{i,k}$ is the same in each period.

Let h_m be the number with lifetime equal to m (number of people with rank i whose eventual lifetime will be m in rank i).

Then $h_1 = n_1$; i.e. all individuals entering period t with lifetime equal to 1. Moreover, $h_2 = n_2 + n_2 = 2n_2$, i.e. all individuals with $l_{i,k} = 2$ that joined in periods t-1 and t. Earlier arrivals with $l_{ik} = 2$ have already departed. Hence $h_m = mn_m$. h_m is also a partition of S_i according to duration. Thus,

$$S_i = \sum_{m=1}^{\infty} h_m = \sum_{m=1}^{\infty} mn_m$$

and $$l_i = \frac{1}{s_i - f_{ii}} \sum_{m=1}^{\infty} mn_m$$

$$= \frac{s_i}{s_i - f_{ii}}$$

Let $q_{ii} = f_{ii}/s_i$.

Then

$$l_i = \frac{1}{(1 - q_{ii})}$$

In this case l_i can be approximated from data as shown in Table 4 on page 15.

(3) Data on the average fraction of people leaving the Air Force each year ($a_{i,t}$), can be obtained from past records. If forced attrition is necessary, the manpower planner can determine reasonable limits on the extent of the attrition.

(4) Data on the number of people/student that can be accommodated or afforded each year ($X_{j,t}$) should be based on the available training facilities and budget allocated, the training department can give this data from their past records.

(5) Data on the average fraction of students dropping out of each school ($\lambda_{j,t}$) can be estimated from past records.

(6) Data on the proportion among ranks (p_i's), should be referred to Department of Defense manpower planning program as described in Table 1 and Table 2 on page 10 and page 11.

(7) Data on the minimum manpower requirement of the Air Force (C) should be based on the organization requirement at that time.

(8) Data on cost of each personnel in a certain rank each year (c_i) can be roughly approximated by the following method:

At year t we paid $k_{i,t}$ dollars for total number of people with rank i. Then

$$c_i = \frac{1}{T} \sum_{t=1}^{T} \frac{k_{i,t}}{S_{i,t}}$$

Notice for the purpose of this discussion, we ignored the cost of medical, housing and etc.

(9) Data on cost of education/training for each type of student (e_j) can be obtained from Training department which is usually in the personnel training index (it already includes the cost of training materials, facilities, living allowance, transportation, accommodation and instructors, as well as training aids).

(10) Data on average attrition cost of each personnel [g_i] may be the most difficult to estimate. This is done basically under some reasonable assumption, such as that we should weigh the cost of attrition of the higher rank greater than the lower rank since the value or the quality of the higher rank is more than the lower rank (knowledge and experience as a consideration). Notice here we don't include the retirement cost (since it is under different budget of another government institution).

(11) Data on the personnel budget allocated each year [B_t] can be obtained from PPBS Department

(12) Data on the number of people entering the Air Force each year is based on the previous record; or can be made by the manpower planner if necessary.

B. EXAMPLE OF MODEL I APPLICATION

Suppose in the beginning of 1978 (call it year 0) the stock level of the Indonesian Air Force is shown in Table 6 and Figure 4 on page 7 .

TABLE 6 The stock level in year 0

Rank	Symbol	Number of People	Note
Rank 1	$S_{1,0}$	3000	Corporal's
Rank 2	$S_{2,0}$	2000	5000
Rank 3	$S_{3,0}$	4000	
Rank 4	$S_{4,0}$	3500	
Rank 5	$S_{5,0}$	3200	
Rank 6	$S_{6,0}$	3500	NCO's
Rank 7	$S_{7,0}$	2500	18700
Rank 8	$S_{8,0}$	1000	
Rank 9	$S_{9,0}$	500	
Rank 10	$S_{10,0}$	2500	
Rank 11	$S_{11,0}$	2000	Officers
Rank 12	$S_{12,0}$	1500	8000
Rank 13	$S_{13,0}$	1500	
Rank 14	$S_{14,0}$	500	
		31,200	

We want in the beginning of 1983 (year 5) the force structure to be well-balanced, that is to satisfy the following conditions:

1. Total number of corporals is around 40% of the total population.
2. Total number of NCO's is around 50% of the total population.
3. Total number of officers is around 10% of the total population.
4. The proportion among corporals, NCO's and officers also should be well-balanced (look at the stock constraints in the next discussion).
5. Total number of people during this transition period should be not less than the minimum manpower requirement, i.e. $\geq 30{,}000$
6. The well-balanced force structure at year 5 should be achieved with the minimum number of people (i.e., force size at year 5 as small as possible).

In achieving this goal, several constraints should be handled properly, otherwise the solution will be infeasible and not relevant to the real world situation.

Based on this situation, then we can set up Model I as follows:

Objective function:

$$\text{Minimize} \quad \sum_{i=1}^{14} S_{i,t} \qquad t = 5$$

subject to:

(1) Flow constraints

$$
\begin{array}{llllllll}
S_{1,1} + a_{1,0} + Z_{1,0} - X_{1,0} & & & & & & = & S_{1,0} \\
S_{1,2} + a_{1,1} + Z_{1,1} - X_{1,1} - S_{1,1} & & & & & & = & 0 \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & & & & & & & \vdots \\
S_{1,5} + a_{1,5} + Z_{1,4} - X_{1,4} - S_{1,4} & & & & & & = & 0 \\
\\
S_{2,1} + a_{2,0} - Z_{1,0} + 0.95X_{2,0} & & & & & & = & S_{2,0} \\
S_{2,2} + a_{2,1} - Z_{1,1} + 0.95X_{2,1} - S_{2,1} & & & & & & = & 0 \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & & & & & & & \vdots \\
S_{2,5} - a_{2,4} - Z_{1,4} + 0.95X_{2,4} - S_{2,4} & & & & & & = & 0 \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & & & & & & & \\
\\
S_{14,1} + a_{14,0} - Z_{13,0} + Z_{14,0} & & & & & & = & S_{14,0} \\
S_{14,2} + a_{14,1} - Z_{13,1} + Z_{14,1} - S_{14,1} & & & & & & = & 0 \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & & & & & & & \vdots \\
S_{14,5} + a_{14,4} - Z_{13,4} + Z_{14,4} - S_{14,4} & & & & & & = & 0
\end{array}
$$

where: $S_{i,0}$'s are known stock level (Table 6)

$\lambda_{j,t}$ is assumed equal $0.05X_{j,t}$ t, j=2,3,4,6.

(2) <u>Policy constraints</u>

(a) Promotion policies

$0.33S_{1,t}$	$\leq$	$Z_{1,t}$	$\leq$	$0.5S_{1,t}$	Promotion from Rank 1 Rank 2
$0.21S_{2,t}$	$\leq$	$X_{2,t}$	$\leq$	$0.35S_{2,t}$	Rank 2 → Rank 3
$0.2S_{3,t}$	$\leq$	$Z_{3,t}$	$\leq$	$0.33S_{3,t}$	Rank 3 → Rank 4
$0.2S_{4,t}$	$\leq$	$Z_{4,t}$	$\leq$	$0.33S_{4,t}$	Rank 4 → Rank 5
$0.2S_{5,t}$	$\leq$	$Z_{5,t}$	$\leq$	$0.33S_{5,t}$	Rank 5 → Rank 6
$0.11S_{6,t}$	$\leq$	$X_{3,t}$	$\leq$	$0.21S_{6,t}$	Rank 6 → Rank 7
$0.2S_{7,t}$	$\leq$	$Z_{7,t}$	$\leq$	$0.33S_{7,t}$	Rank 7 → Rank 8
$0.11S_{8,t}$	$\leq$	$X_{4,t}$	$\leq$	$0.21S_{8,t}$	Rank 8 → Rank 9
$0.25S_{9,t}$	$\leq$	$Z_{9,t}$	$\leq$	$0.5S_{9,t}$	Rank 9 → Rank 10
$0.25S_{10,t}$	$\leq$	$Z_{10,t}$	$\leq$	$0.5S_{10,t}$	Rank 10 → Rank 11
$0.14S_{11,t}$	$\leq$	$Z_{11,t}$	$\leq$	$0.33S_{11,t}$	Rank 11 → Rank 12
$0.13S_{12,t}$	$\leq$	$X_{6,t}$	$\leq$	$0.26S_{12,t}$	Rank 12 → Rank 13
$0.13S_{13,t}$	$\leq$	$Z_{13,t}$	$\leq$	$0.25S_{13,t}$	Rank 13 → Rank 14
$0.1S_{14,t}$	$\leq$	$Z_{14,t}$	$\leq$	$0.25S_{14,t}$	Rank 14 → Rank 15

(b) Recruitment policies

$20 \le Y_{11,t} \le 50$ new input to rank 11/year

$X_{1,t} \le 3000$ new input to rank 1/year

$80 \le X_{5,t} \le 200$ new input to rank 10/year

Note: $X_{j,t}$ means promotion with training requirement for $j = 2,3,4,6$.

(c) Attrition policies

$a_{1,t} \le 0.01S_{1,t}$	$t=0,1,\ldots,4$	attrition from rank 1 each year	
$a_{2,t} \le 0.02S_{2,t}$		attrition from rank 2 each year	
$a_{3,t} \le 0.03S_{3,t}$		attrition from rank 3 each year	
$a_{4,t} \le 0.04S_{4,t}$		attrition from rank 4 each year	
$a_{5,t} \le 0.08S_{5,t}$		attrition from rank 5 each year	
$a_{6,t} \le 0.1S_{6,t}$		attrition from rank 6 each year	
$a_{7,t} \le 0.2S_{7,t}$		attrition from rank 7 each year	
$a_{8,t} \le 0.3S_{8,t}$		attrition from rank 8 each year	
$a_{9,t} \le 0.3S_{9,t}$		attrition from rank 9 each year	
$a_{10,t} \le 0.1S_{10,t}$		attrition from rank 10 each year	

$a_{11,t} \leq 0.1S_{11,t}$ — attrition from rank 11 each year

$a_{12,t} \leq 0.1S_{12,t}$ — attrition from rank 12 each year

$a_{13,t} \leq 0.1S_{13,t}$ — attrition from rank 13 each year

$a_{14,t} \leq 0.25S_{14,t}$ — attrition from rank 14 each year

Note: These attrition policies are based on past data, where usually the NCO's start to retire at rank 7 and the officers at rank 14.

(3) Stock constraints

(a) Total stock constraint

$$\sum_{i=1}^{14} S_{i,t} \geq 30{,}000 \qquad t=1,2,\ldots,5$$

minimum manpower requirement

(b) Ratio among group ranks

$$0.35 \sum_{i=1}^{14} S_{i,t} \leq \sum_{i=1}^{2} S_{i,t} \leq 0.45 \sum_{i=1}^{14} S_{i,t} \qquad t=5$$

Total corporals constraint

$$0.45 \sum_{i=1}^{14} S_{i,t} \leq \sum_{i=3}^{9} S_{i,t} \leq 0.55 \sum_{i=1}^{14} S_{i,t} \qquad t=5$$

Total NCO's constraint

$$0.10 \sum_{i=1}^{14} S_{i,t} \leq \sum_{i=10}^{14} S_{i,t} \leq 0.15 \sum_{i=1}^{14} S_{i,t} \qquad t=5$$

Total officers constraint

(c) Ratio within officers rank

$$0.18 \sum_{i=10}^{14} S_{i,t} \le S_{10,t} \le 0.22 \sum_{i=10}^{14} S_{i,t} \quad t=5 \quad \text{rank 10 constraint}$$

$$0.13 \sum_{i=10}^{14} S_{i,t} \le S_{11,t} \le 0.17 \sum_{i=10}^{14} S_{i,t} \quad t=5 \quad \text{rank 11 constraint}$$

$$0.24 \sum_{i=10}^{14} S_{i,t} \le S_{12,t} \le 0.28 \sum_{i=10}^{14} S_{i,t} \quad t=5 \quad \text{rank 12 constraint}$$

$$0.19 \sum_{i=10}^{14} S_{i,t} \le S_{13,t} \le 0.23 \sum_{i=10}^{14} S_{i,t} \quad t=5 \quad \text{rank 13 constraint}$$

$$0.14 \sum_{i=10}^{14} S_{i,t} \le S_{14,t} \le 0.18 \sum_{i=10}^{14} S_{i,t} \quad t=5 \quad \text{rank 14 constraint}$$

(d) Ratio within NCO's

$$0.26 \sum_{i=3}^{9} S_{i,t} \le S_{3,t} \le 0.30 \sum_{i=3}^{9} S_{i,t} \quad t=5 \quad \text{rank 3 constraint}$$

$$0.20 \sum_{i=2}^{9} S_{i,t} \le S_{4,t} \le 0.24 \sum_{i=3}^{9} S_{i,t} \quad t=5 \quad \text{rank 4 constraint}$$

$$0.16 \sum_{i=3}^{9} S_{i,t} \le S_{5,t} \le 0.18 \sum_{i=3}^{9} S_{i,t} \quad t=5 \quad \text{rank 5 constraint}$$

$$0.12 \sum_{i=3}^{9} S_{i,t} \leq S_{6,t} \leq 0.14 \sum_{i=3}^{9} S_{i,t} \qquad t=5 \qquad \text{rank 6 constraint}$$

$$0.09 \sum_{i=3}^{9} S_{i,t} \leq S_{7,t} \quad 0.11 \sum_{i=3}^{9} S_{i,t} \qquad t=5 \qquad \text{rank 7 constraint}$$

$$0.06 \sum_{i=3}^{9} S_{i,t} \leq S_{8,t} \leq 0.08 \sum_{i=3}^{9} S_{i,t} \qquad t=5 \qquad \text{rank 8 constraint}$$

$$0.03 \sum_{i=3}^{9} S_{i,t} \leq S_{9,t} \leq 0.05 \sum_{i=3}^{9} S_{i,t} \qquad t=5 \qquad \text{rank 9 constraint}$$

(e) Ratio within corporals

$$0.55 \sum_{i=1}^{2} S_{i,t} \leq S_{i,t} \leq 0.60 \sum_{i=1}^{2} S_{i,t} \qquad t=5 \qquad \text{rank 1 constraint}$$

Note: all of these stock constraints except the total stock constraint are applied only at year 5, in order to give more flexibility to the model when adjusting the stock level during this transition period from the unbalanced force structure to the well-balanced force structure

(4) Budget Constraint

$$\sum_{i=1}^{14} C_i S_{i,t+1} + \sum_{i=1}^{14} g_i a_{i,t} + \sum_{j=1}^{6} e_j X_{j,t} \leq B_t$$

The coefficients of C_i's, g_i's, e_j's and B_t are given as follows:

(a) Average salary of rank i/year:

C_1 = \$578.

C_2 = 650.

C_3 = 867.

C_4 = 940.

C_5 = 968.

C_6 = 1012.

C_7 = 1156.

C_8 = 1230.

C_9 = 1257.

C_{10} = 1879.

C_{11} = 2024.

C_{12} = 2313.

C_{13} = 2891.

C_{14} = 3180.

(b) Average attrition cost of rank i/year

$g_1 = \$144$

$g_2 = 173$

$g_3 = 289$

$g_4 = 314$

$g_5 = 322$

$g_6 = 337$

$g_7 = 385$

$g_8 = 410$

$g_9 = 419$

$g_{10} = 628$

$g_{11} = 674$

$g_{12} = 771$

$g_{13} = 943$

$g_{14} = 1060$

C) Cost of training per person in school j/year

$e_1 = \$800$

$e_2 = 1200$

$e_3 = 800$

$e_4 = 1500$

$e_5 = 1700$

$e_6 = 2000$

(d) Budget available, we assume in this case each year is the same

$$B_t = \$42,000,000 \qquad \text{for all } t$$

2. The size of Linear programming problem of Model I in this case is 348 constraints and 581 variables, it took about 4 minutes cpu time to solve the problem by the mathematical programming package MPS-360 (data input consisted of 1617 cards).

The input and output of this computer program is not attached in this paper since it is just a regular programming routine. However for the purpose of showing the solution of the problem, the output of the program has been extracted into graphical form as illustrated in Figure 4 through Figure 9.

From these pictures we can observe the movement or the adjustment of the force structure throughout the years. The manpower planner will be able to check whether their policies (constraints) have created a good or bad influence toward their goal. If it is necessary they can adjust their policy in such a way, until they get a reasonable solution. Based on the solution as shown in Figure 9, we can conclude that Model I has worked pretty well.

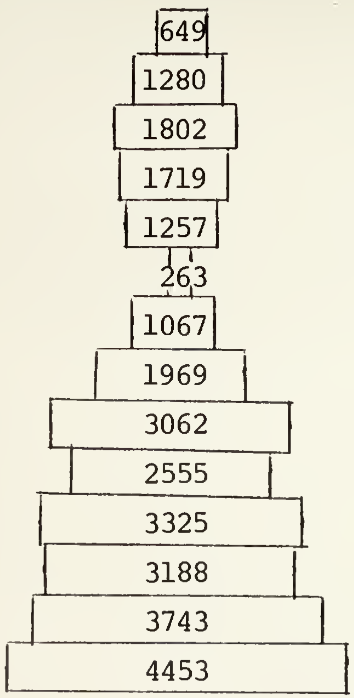
Rank i	YEAR 0	New input	$z_{i,0}$		$a_{i,0}$	
14	500		125	0.25	125	0.25
13	1500		375	0.25	150	0.1
12	1500		371	0.106	150	0.1
11	2000	20	660	0.33	160	0.08
10	2500	80	1017	0.41	125	0.05
9	500		125	0.25	150	0.3
8	1000		104	0.10	300	0.3
7	2500		500	0.2	500	0.2
6	3500		574	0.16	350	0.1
5	3200		1056	0.33	256	0.08
4	3500		700	0.2	140	0.04
3	4000		800	0.2	120	0.03
2	2000		399	0.2	40	0.02
1	3000	1421	990	0.33	30	0.01
Total ⟶ Number of people in the system	31200	1521 ↑ Total Number of new input			2596 ↑ Total number who leave the system	

FIGURE 4

Rank i	YEAR 1	New input	$z_{i,1}$		$a_{i,1}$	
14	625		156	0.25	156	0.25
13	1346		336	0.25	135	0.1
12	1640		405	0.25	164	0.1
11	2217	20	732	0.33	177	0.08
10	1563	80	391	0.25	78	0.05
9	330		82	0.25	99	0.3
8	1096		115	0.1	329	0.3
7	2074		415	0.2	415	0.2
6	3632		725	0.2	363	0.1
5	2588		518	0.2	207	0.03
4	3460		692	0.2	138	0.04
3	3479		696	0.2	104	0.03
2	2551		509	0.2	0	0.
1	3401	2786	1701	0.5	34	0.01
Total	30002	2886			2399	

FIGURE 5

YEAR 2

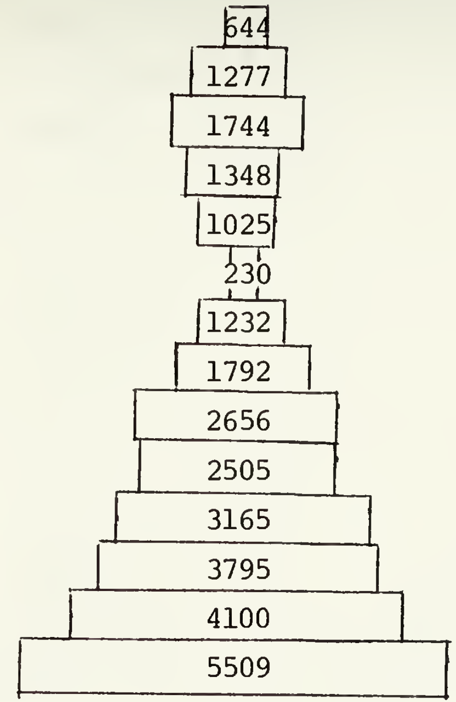


	$z_{i,2}$		$a_{i,2}$	
	162	0.25	162	0.25
	320	0.25	128	0.1
	446	0.25	180	0.1
20	567	0.33	137	0.08
80	314	0.25	63	0.05
	66	0.25	79	0.3
	111	0.1	118	0.11
	394	0.2	394	0.2
	611	0.2	306	0.1
	511	0.2	204	0.08
	665	0.2	133	0.04
	638	0.2	0	0.
	1245	0.33	0	0.
2658	1601	0.4	0	0.

30332 2758 1904

FIGURE 6

YEAR 3

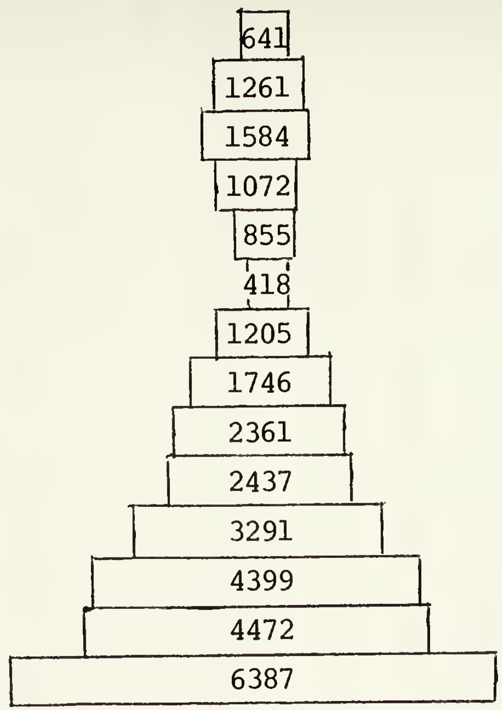


	$z_{i,3}$		$a_{i,3}$	
	161	0.25	161	0.25
	319	0.25	128	0.1
	430	0.25	174	0.1
20	445	0.33	108	0.08
80	256	0.25	51	0.05
	57	0.25	0	0.
	246	0.2	139	0.11
	358	0.2	217	0.12
	530	0.2	266	0.1
	501	0.2	200	0.08
	633	0.2	0	0.
	759	0.2	0	0.
	1363	0.33	82	0.02
2696	1818	0.33	0	

31022 2796 1526

FIGURE 7

YEAR 4

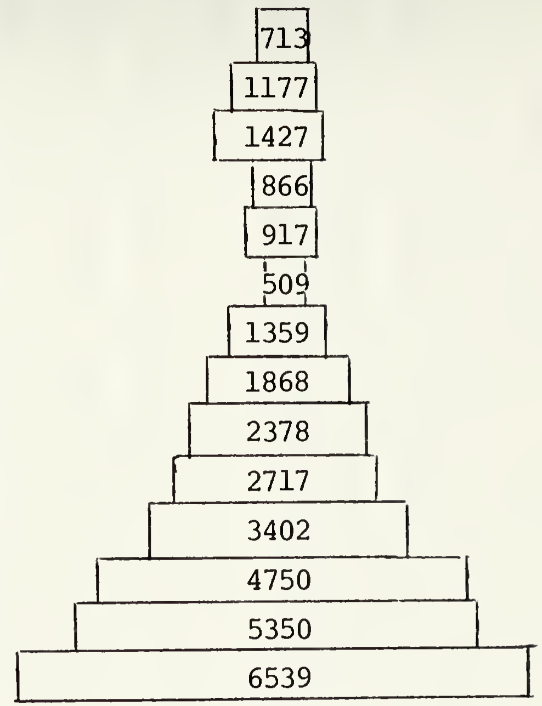


32129

FIGURE 8

	$z_{i,4}$		$a_{i,4}$	
	160	0.25	83	0.13
	315	0.25	126	0.1
	353	0.22	158	0.1
50	354	0.33	86	0.08
171	214	0.25	0	
	105	0.25	0	
	196	0.16	0	
	349	0.2	0	
	471	0.2	0	
	487	0.2	0	
	768	0.23	0	
	880	0.2	0	
	1230	0.28	0	
2259	2108	0.33	0	
2480			453	

YEAR 5



33966

FIGURE 9

C. EXAMPLE OF MODEL II APPLICATION

Given a well-balanced force structure as shown on Figure 9, we want to maintain it for another 5 year period. For this example let us use the data from the solution of Model I at year 5 as exhibited in Figure 9 as a starting state.

As has been mentioned in the previous section, the linear programming model used in Model II is almost the same as Model I, except some modifications are made in the stock constraints. Then the linear programming model of Model II is:

$$\text{Minimize} \quad \sum_{i=1}^{14} S_{i,t} \qquad t = 5$$

Subject to:

(1) Flow constraints (same as Model I)

(2) Policy constraints (same as Model I)

(3) Budget constraints (same as Model I)

(4) Stock constraints

Notice in stock constraints some modification is made such as:

a. instead of $t = 5$ as in Model I, now $t = 1,2,\ldots,5$

b. Corporal's rank proportion constraint is not used in this model (for further explanation see page 39 or Figure 15a and 15b in the following discussion).

c. The lieutenant's rank proportion constraint such as rank 10 and rank 11 constraint as shown in Model I, is put together into one constraint in this model, i.e.

$$0.31 \sum_{i=10}^{14} S_{i,t} \leq \sum_{i=10}^{11} S_{i,t} \leq 0.39 \sum_{i=10}^{14} S_{i,t}$$

$$t = 1,2,\ldots,5$$

We apply the stock constraints throughout the years, i.e. $t = 1,2,\ldots,5$, in order to prevent the force structure from changing wildly each year as it happens in Model I, which is obviously undesirable for a steady state well-balanced force structure.

2. By using the MPS-360 package, this linear programming problem which now has a size of 456 constraints and 702 variables (about 2248 cards), can be solved within 10 minutes CPU time, a little bit longer than Model I.

The solution of this problem is also extracted in graphical form as exhibited in Figure 10 up to Figure 15. Notice that, by eliminating the corporal's rank proportion constraint, the shape of the force structure becomes slightly different in the bottom part (i.e. rank 1 is smaller than rank 2 in size), which is usually unexpected for a real well-balanced structure. In fact we can solve this problem by applying the corporal's rank proportion constraint to the

model, but it will create a significant increase in the force size every year due to large inputs needed to maintain this corporal's rank ratio. Furthermore, if we let this situation exist throughout the years, eventually the solution will be infeasible, except if we made another modification to the constraints in such a way to make the solution become feasible again. Figure 15a and Figure 15b on page 76 show the solution of the model when not applying and applying this constraint.

Thus it appears that the corporal's rank proportion constraint is inconsistent with our planned promotion policies and total force level goals. For this reason it has been left out of Model II and the resulting imbalance situation in corporal's rank is accepted as necessary. (In fact, based on past experience the problem in this corporal's rank ratio is not very critical as compared to the problem in the force size).

Regardless of these problems, if we look to the movements of the flow in the system throughout the years, this model is quite successful in maintaining the balance of the force structure, although not perfectly well. As in Model I, in this model the manpower planners still have freedom to adjust their policy (i.e. constraints) to improve their solution from this problem.

YEAR 5

		$z_{i,5}$		$a_{i,5}$	
713		178	0.25	178	0.25
1171		293	0.25	117	0.1
1427		280	0.2	143	0.1
866	20	247	0.29	87	0.1
917	80	459	0.5	92	0.1
509		127	0.25	153	0.3
1359		258	0.19	408	0.3
1868		505	0.27	374	0.2
2378		474	0.2	238	0.1
2717		610	0.22	217	0.08
3402		1036	0.3	136	0.04
4750		1449	0.3	143	0.03
5350		1068	0.2	107	0.02
6539	540	2158	0.33	65	0.01
33966	640			2458	

FIGURE 10

YEAR 6

		$z_{i,6}$		$a_{i,6}$	
650		163	0.25	163	0.25
1041		260	0.25	104	0.1
1251		263	0.21	125	0.1
1011	20	267	0.26	81	0.08
573	80	287	0.5	29	0.05
488		122	0.25	122	0.25
1197		238	0.2	359	0.3
1463		365	0.25	105	0.07
2276		454	0.2	228	0.1
2926		656	0.22	234	0.08
3679		858	0.23	147	0.04
4226		1184	0.28	127	0.03
6333		1264	0.2	127	0.2
4856	1103	1603	0.33	49	0.01
31970	1203			2000	

FIGURE 11

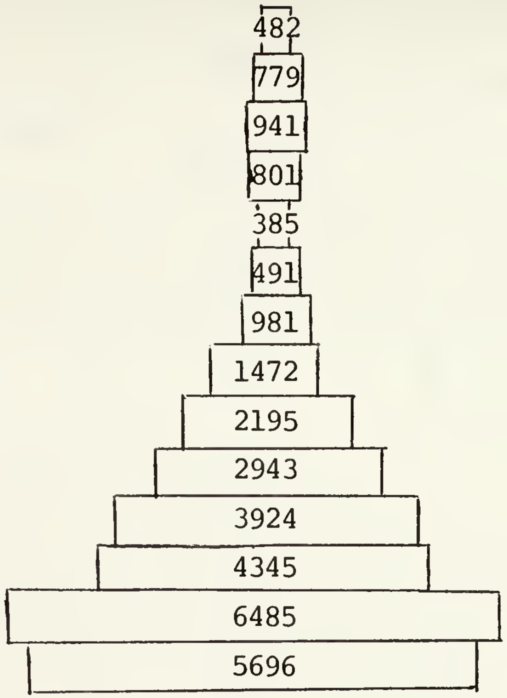
YEAR 7		$z_{i,7}$		$s_{i,7}$	
585		136	0.23	146	0.25
939		235	0.25	94	0.1
1130		244	0.22	113	0.1
970	20	249	0.26	78	0.08
460	80	230	0.5	23	0.05
482		121	0.25	0	0.
964		125	0.13	289	0.3
1447		423	0.3	13	0.01
2250		449	0.2	225	0.1
2893		695	0.25	231	0.08
3858		953	0.25	154	0.04
4179		1142	0.27	125	0.03
6546		1306	0.2	131	0.02
4309	1360	1422	0.33	43	0.01
31012	1460			1665	

FIGURE 12

YEAR 8		$z_{i,8}$		$a_{i,8}$	
538		134	0.25	134	0.25
854		214	0.22	85	0.1
1023		224	0.22	102	0.1
893	20	244	0.27	71	0.08
407	80	204	0.5	20	0.05
487		122	0.25	69	0.14
973		193	0.2	90	0.09
1460		292	0.2	149	0.1
2271		453	0.2	227	0.1
2920		605	0.21	234	0.08
3893		862	0.22	156	0.04
4218		1049	0.25	127	0.03
6531		1302	0.2	131	0.02
4204	2921	1387	0.33	42	0.01
30672	3021			1637	

FIGURE 13

YEAR 9

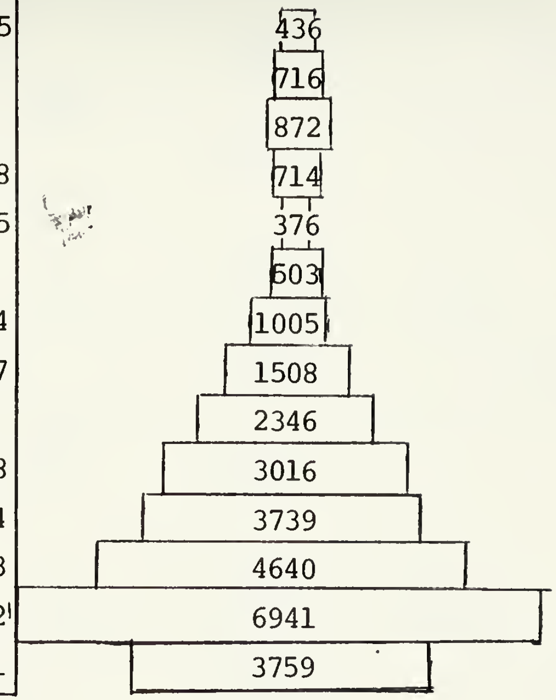


31920

FIGURE 14

	$z_{i,9}$		$a_{i,9}$	
	121	0.25	121	0.25
	195	0.25	78	0.1
	210	0.22	94	0.1
20	235	0.3	64	0.08
80	193	0.5	19	0.05
	123	0.25	0	0
	135	0.14	135	0.14
	294	0.2	107	0.07
	438	0.2	0	0
	589	0.2	235	0.08
	397	0.23	157	0.04
	369	0.2	130	0.03
	1294	0.2	130	0.02
	1880	0.33	57	0.01
100			1327	

YEAR 10



30571

FIGURE 15

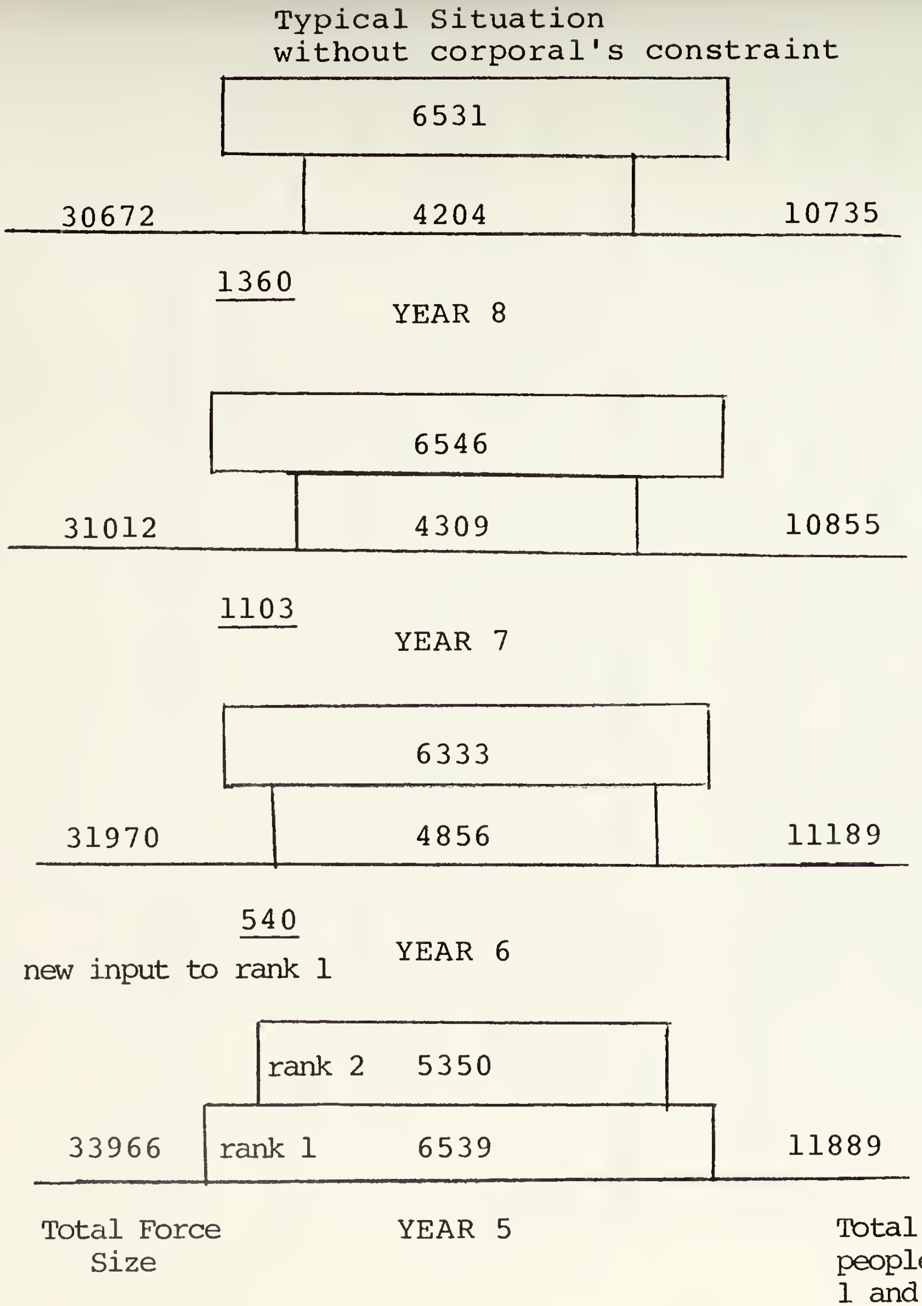


FIGURE 15a

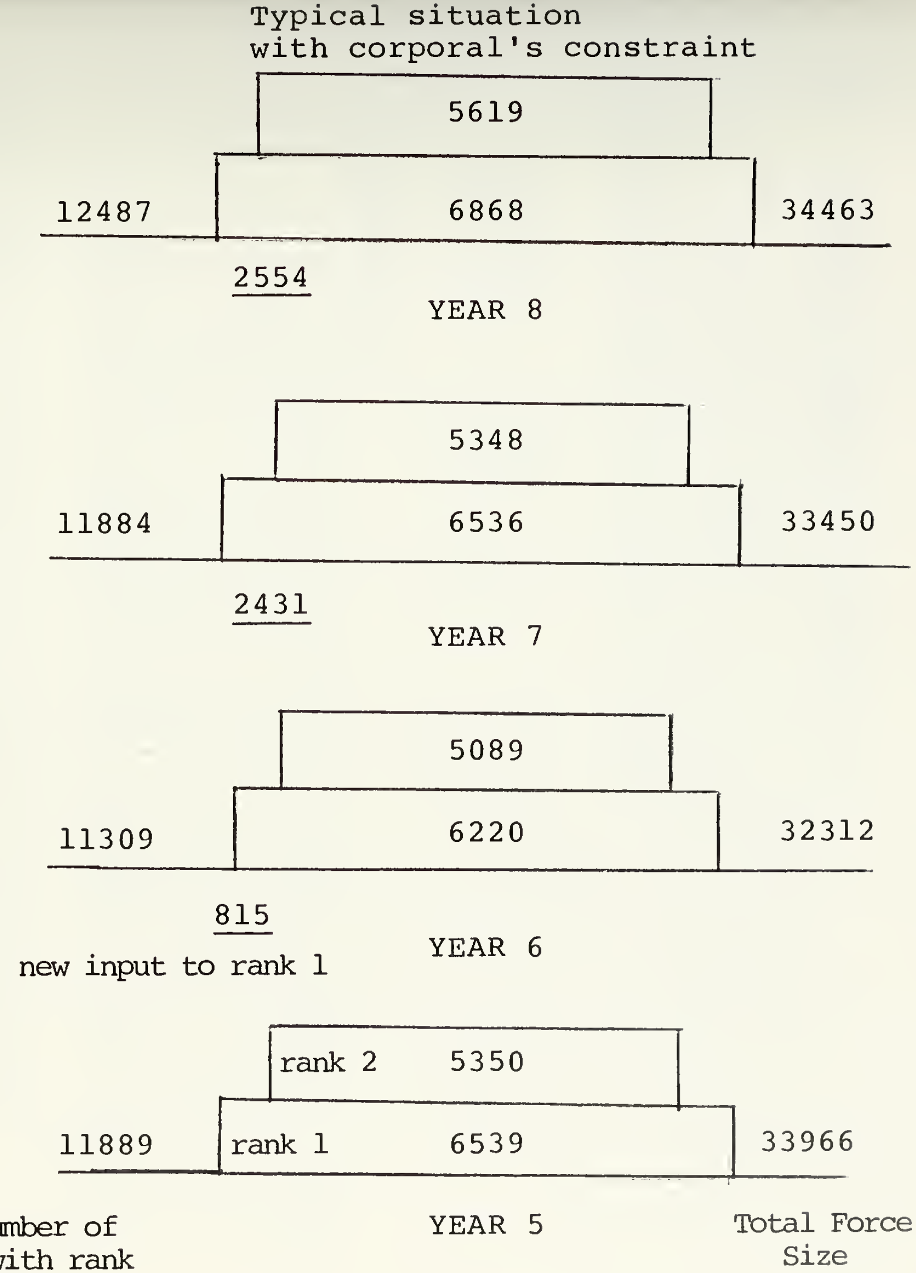


FIGURE 15b

D. EXAMPLE OF MODEL III APPLICATION

The problem in example C i.e. maintaining the well-balanced force structure can be solved by Model III by the following linear programming method

$$\text{Minimize} \quad \sum_{i=1}^{14} S_{i,t} \qquad t = 1$$

Subject to:

(1) Flow constraint (same as Model II, but t = 0,1)

(2) Policy constraint (same as Model II, but t = 0)

(3) Budget constraint (same as Model II, but t = 0)

(4) Stock constraint (same as Model II, but t = 1)

If in Model II we solved the 5 years planning period simultaneously, in Model III we solved it year by year. Hence this model can be used for 1 year planning period or more; and the linear program consequently will be smaller than the previous model.

2. Using the same MPS-360 package, this linear programming which has a size of 106 constraints and 170 variables (about 524 cards), can be solved within less than 21 seconds. The solution of this problem is exhibited in Figure 16 up to Figure 21. If we compare this to the solution of Model II, the result of this Model III is almost the same, only slight differences exist.

The advantages of using Model III as compared to Model II are:

a. The manpower planner will have more flexibility in adjusting their policy if they need to improve the solution.

b. The manpower planner will find it easier to trace the impact of their new policy toward the solution.

c. It can be solved in a smaller computer when the large computer is not available.

However, the disadvantage of using this model is that it needs some manual computation every time a one year planning program is run.

E. EXAMPLE OF MODEL IV APPLICATION

Another alternative for solving the problem as stated in example C (maintaining the well-balanced force structure) is by using a cross-sectional model as has been defined in the previous discussion. Assuming that the manpower planners are already satisfied with the solution they obtained from Model III, then in order to simplify the computation of the force structure in the future, they might try to use Model IV for this purpose. But before they decide to use it extensively, it is better to check first whether the flow coefficients which the model uses are already suitable or not for achieving their goal. This can be done by trying to use this model to solve problem C (with the same data to start) and then consider its solution.

Suppose in this case the recruitment, promotion and attrition policy used is obtained by averaging the solution from Model III, i.e.

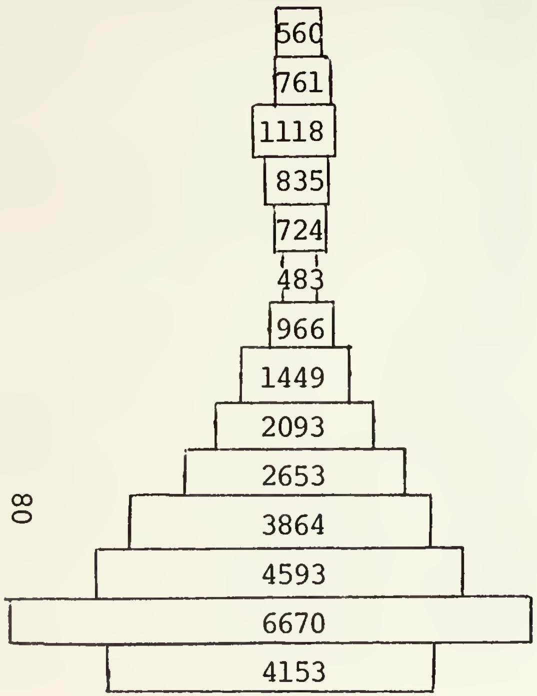
YEAR 5		$z_{i,5}$		$a_{i,5}$	
713		178	0.25	178	0.25
1171		293	0.25	117	0.1
1427		177	0.12	143	0.1
866	50	121	0.14	87	0.1
917	80	229	0.25	92	0.1
509		127	0.25	153	0.3
1359		258	0.2	408	0.3
1868		374	0.2	374	0.2
2378		474	0.2	238	0.1
2717		579	0.21	217	0.08
3402		680	0.2	136	0.04
4750		1316	0.28	143	0.03
5350		1068	0.2	107	0.02
6539	530	2158	0.33	65	0.01
33966	660		2458		

FIGURE 16

YEAR 6		$z_{i,6}$		$a_{i,6}$	
649		161	0.25	162	0.25
937		234	0.25	94	0.1
1229		152	0.12	123	0.1
887	50	164	0.18	89	0.1
803	80	201	0.25	80	0.1
488		122	0.25	0	0.
1066		117	0.11	302	0.28
1595		319	0.2	275	0.17
2245		447	0.2	225	0.1
2601		520	0.2	208	0.08
3901		780	0.2	156	0.04
4359		899	0.21	131	0.03
6334		1264	0.2	0	0.
4846	906	1599	0.33	0	0.
31940	1036		1845		

FIGURE 17

YEAR 7



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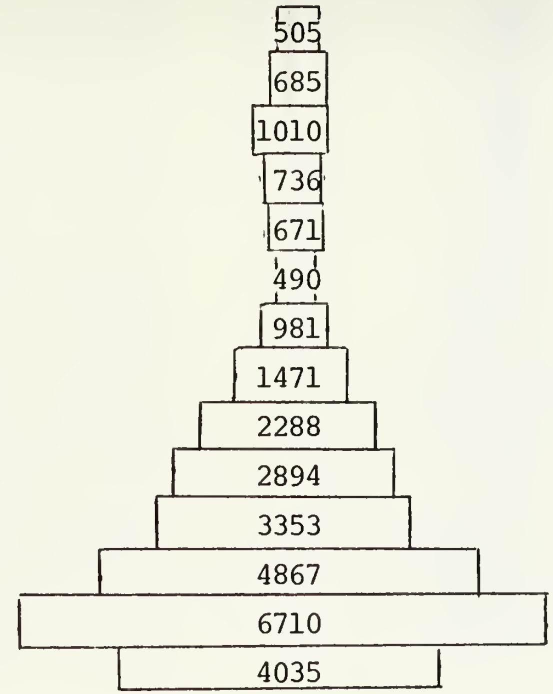
	$z_{i,7}$		$a_{i,7}$	
	105	0.19	140	0.25
	190	0.25	76	0.1
	191	0.17	112	0.1
50	196	0.23	84	0.1
80	181	0.25	72	0.1
	121	0.25	0	0.
	128	0.13	253	0.26
	396	0.27	0	0.
	418	0.2	209	0.1
	822	0.31	212	0.08
	1275	0.33	155	0.04
	919	0.2	138	0.03
	1331	0.2	0	0.
1252	1370	0.33	0	0.
1382			1451	

FIGURE 18

YEAR 8



30697

	$z_{i,8}$		$a_{i,8}$	
	126	0.25	90	0.18
	172	0.25	69	0.1
	250	0.25	101	0.1
50	243	0.33	74	0.1
80	168	0.25	67	0.1
	123	0.25	68	0.14
	196	0.2	235	0.24
	441	0.3	0	0.
	456	0.2	229	0.1
	690	0.24	232	0.08
	671	0.2	134	0.04
	1105	0.23	146	0.03
	1339	0.2	134	0.02
1431	1332	0.33	40	0.01
1561			1619	

FIGURE 19

YEAR 9

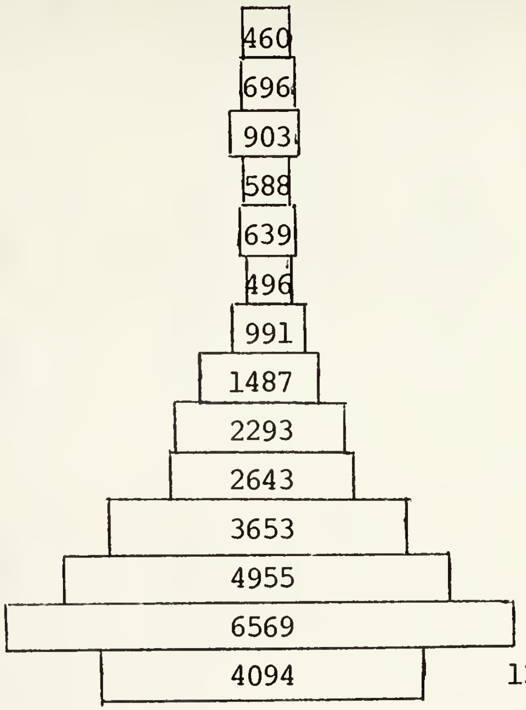


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FIGURE 20

	$z_{i,9}$		$a_{i,9}$	
	74	0.16	115	0.25
	153	0.22	70	0.1
	223	0.25	90	0.1
50	137	0.23	59	0.1
80	160	0.25	54	0.08
	124	0.25	0	0.
	128	0.13	161	0.16
	297	0.2	148	0.1
	458	0.2	229	0.1
	726	0.27	211	0.08
	960	0.26	146	0.04
	1119	0.23	149	0.03
	1310	0.2	0	0.
1249	1351	0.33	0	0.
1379			1432	

YEAR 10

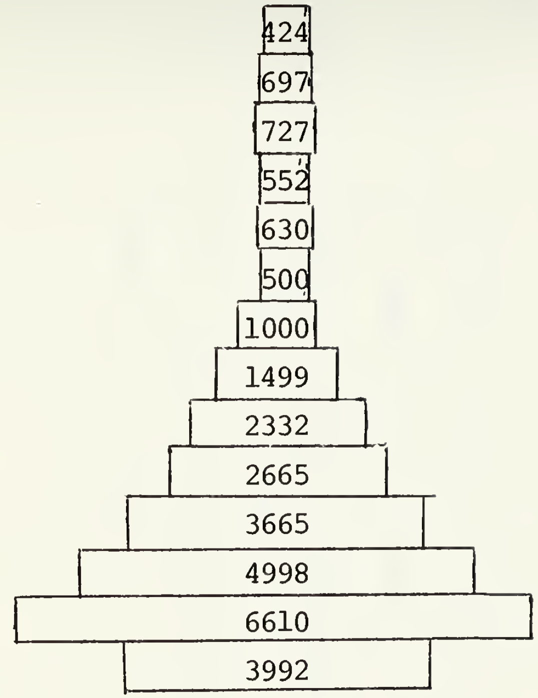

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FIGURE 21

$$q_{i+1,i} = \frac{1}{T} \sum_{t=1}^{T} \frac{Z_{i,t}}{S_{i,t}} \qquad T = 5$$

where

$q_{i+1,i}$ = fraction of flow of people from rank i to rank i+1 each year

$Z_{i,t}$ = number of people promoted from rank i to rank i+1 at year t

$S_{i,t}$ = total number of people with rank i at the beginning of year t

$$q_{ii} = 1 - q_{i+1,i} - \frac{1}{T} \sum_{t=1}^{T} \frac{a_{i,t}}{S_{i,t}} \qquad T = 5$$

where

$q_{i,i}$ = fraction of people who stay at rank i each year

$a_{i,t}$ = number of people who leave the system from rank i at year t

$$f_{0,1} = \frac{1}{T} \sum_{t=1}^{T} X_{1,t} \qquad T = 5$$

$$f_{0.10} = \frac{1}{T} \sum_{t=1}^{T} X_{5,t} \qquad T = 5$$

$$f_{0.11} = \frac{1}{T} \sum_{t=1}^{T} Y_{11,t} \qquad T = 5$$

where:

$f_{0,i}$ = average number of people entering the system to rank i

$X_{1,t}$ = number of new inputs of rank 1 at year t

$X_{5,t}$ = number of new inputs of rank 10 at year t

$Y_{11,t}$ = number of new inputs of rank 11 at year t.

These approximations are indeed rough, but are useful as a starting point which can be adjusted if the solution is not suitable.

Then Model IV in this example will be:

$$S(t+1) = f_0(t) + Q\,S(t)$$

where: $S(t+1)$ = column vector of size 14 which is not known prior to computation

$f_0(t)$ = column vector of size 14, with

$f_{0,1} = 1074$

$f_{0,10} = 80$

$f_{0,11} = 50$

and the rest are zeros

Q = flow matrix of size 14×14 where the value of q_{ij} is shown on Table 7.

$S(t)$ = column vector of size 14; start with data obtained from stock level at year 5 from Model I solution (see Figure 22)

TABLE 7. Non-zero Elements of Flow Matrix Q

$q_{j,i}$	$q_{i,i}$
$q_{2,1} = 0.33$	$q_{1,1} = 0.666$
$q_{3,2} = 0.20$	$q_{2,2} = 0.792$
$q_{4,3} = 0.23$	$q_{3,3} = 0.74$
$q_{5,4} = 0.22$	$q_{4,4} = 0.74$
$q_{6,5} = 0.23$	$q_{5,5} = 0.69$
$q_{7,6} = 0.20$	$q_{6,6} = 0.70$
$q_{8,7} = 0.32$	$q_{7,7} = 0.586$
$q_{9,8} = 0.15$	$q_{8,8} = 0.602$
$q_{10,9} = 0.25$	$q_{9,9} = 0.662$
$q_{11,10} = 0.25$	$q_{10,10} = 0.654$
$q_{12,11} = 0.22$	$q_{11,11} = 0.68$
$q_{13,12} = 0.18$	$q_{12,12} = 0.72$
$q_{14,13} = 0.24$	$q_{13,13} = 0.66$
$q_{15,14} = 0.22$	$q_{14,14} = 0.54$

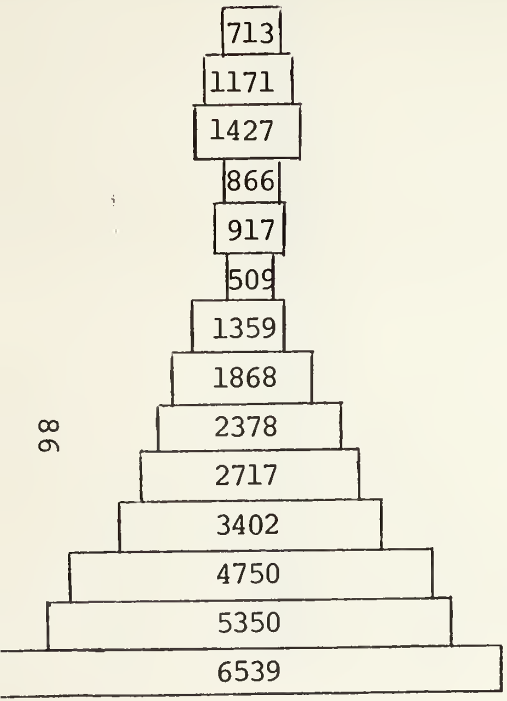
Note: $q_{15,14}$ is not used, and all other elements not mentioned here are all zeros.

2. With a simple algorithm we can solve this problem with Model IV by the computer very fast. For example, for 100 year planning, it only takes 6 seconds CPU time (about 82 cards). What is interesting in this model is that the size of the force structure is decreasing year after year. From 33,966 people (at year 5) at the beginning of planning it becomes 30,336 people at the end of year 10, and it still is decreasing continuously, then it stays at 24,016.7 at year 100 permanently. But the total size of rank 1 becomes constant after year 32 at size 3251.50. (Notice even in this model rank 1 is less than rank 2.) However, we should be aware that this occurrence varies according to the data and the flow parameters we picked. For a short run basis, this model works well enough and is quite easy to compute. The solution of this model for a ten year planning period is shown on Figure 22 up to Figure 27.

F. DISCUSSION OF THE APPLICATION OF THE MODELS

Based on those four examples discussed in the previous section, we can observe how the size of force structure varies each year for each different model used. Figure 28 shows the relation of force structure with respect to time for each model, where for year 0 to year 5 is the transition period and year 5 to year 10 is the steady state period. There is a significant increase of manpower during that transition period, this is mainly because we force the structure to have more rank 1 than rank 2 (to have an ideal well-balanced force structure).

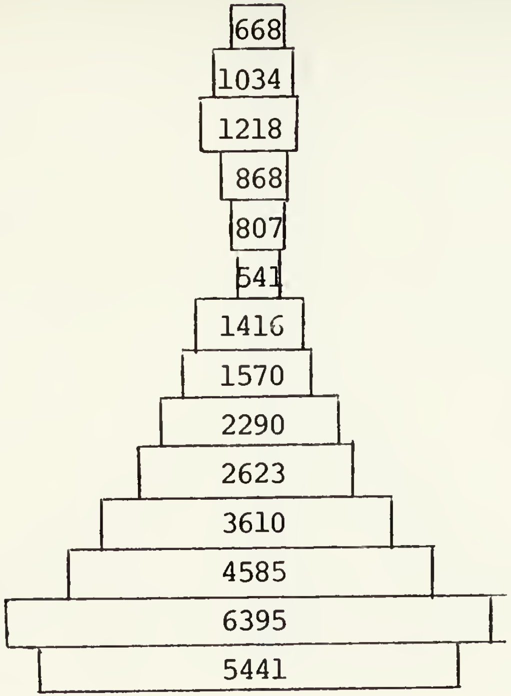
YEAR 5



33966

FIGURE 22

YEAR 6



33065

FIGURE 23

YEAR 7

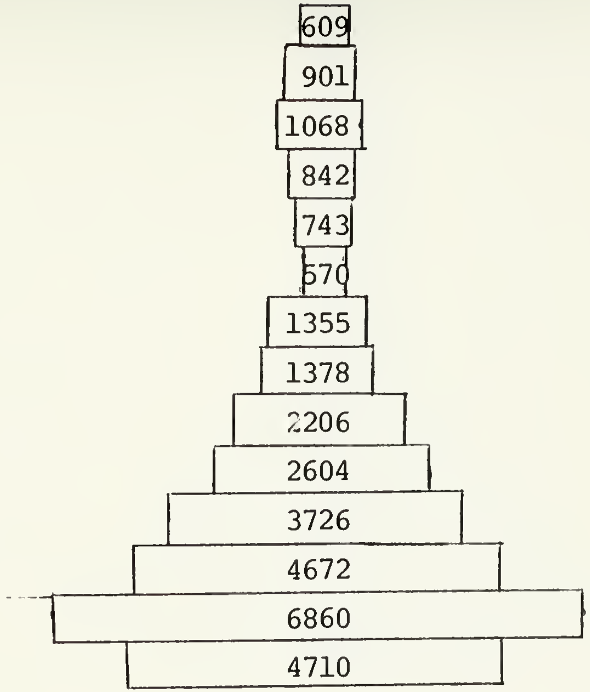


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FIGURE 24

q_{ij}	q_{i0}
0.22	0.24
0.24	0.1
0.18	0.1
0.22	0.1
0.25	0.096
0.25	0.088
0.15	0.248
0.32	0.094
0.2	0.1
0.23	0.08
0.22	0.04
0.23	0.03
0.2	0.008
0.33	0.004

YEAR 8	YEAR 9	YEAR 10

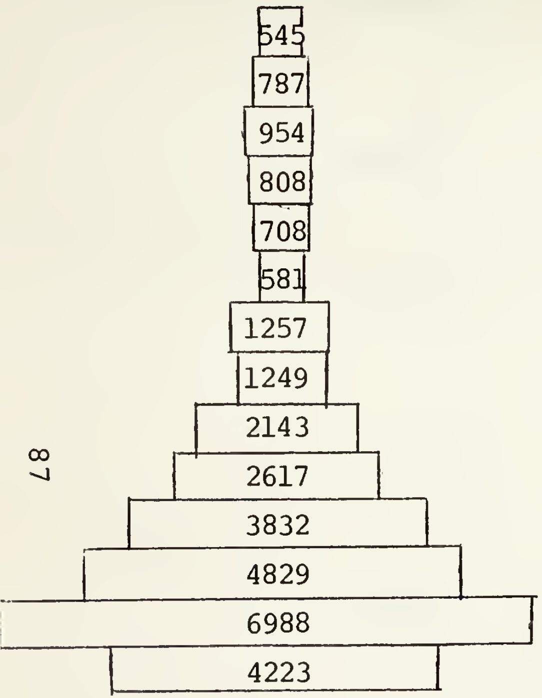

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FIGURE 25

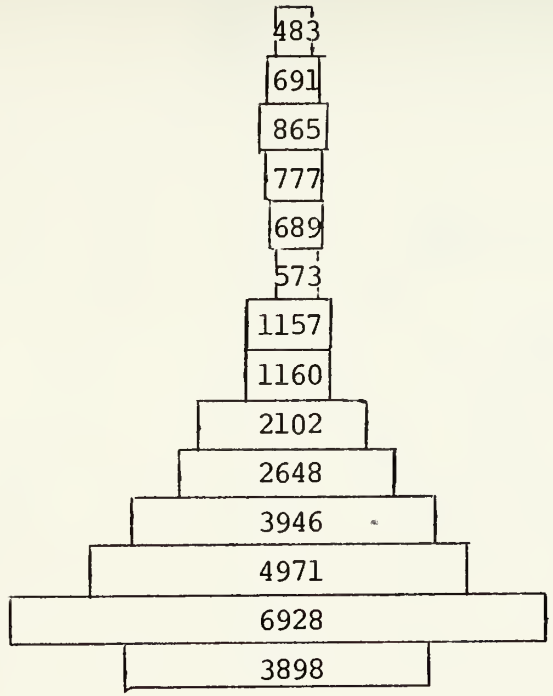

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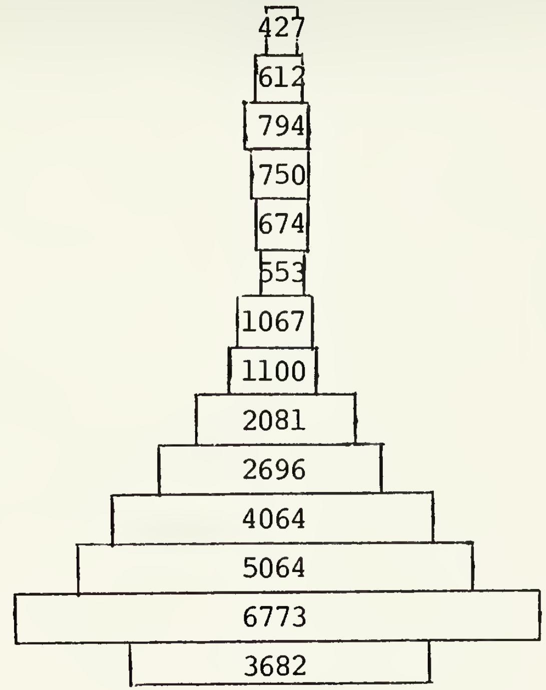
FIGURE 26



30336

FIGURE 27

q_{ij}	q_{i0}
0.22	0.24
0.24	0.1
0.18	0.1
0.22	0.1
0.25	0.096
0.25	0.088
0.15	0.248
0.32	0.094
0.2	0.1
0.23	0.08
0.22	0.04
0.23	0.03
0.2	0.008
0.33	0.004

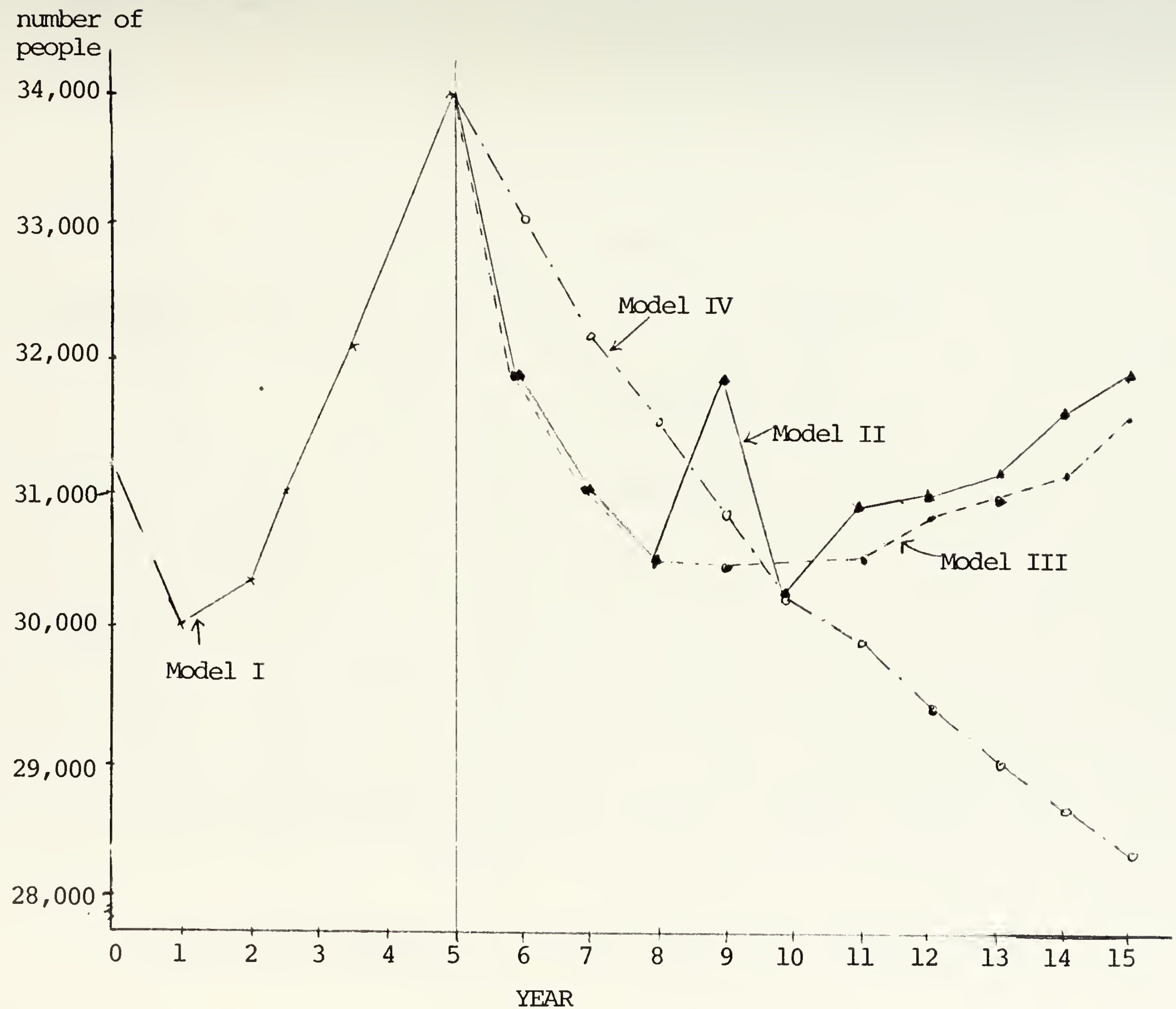


FIGURE 28. Movement of Total Force Size

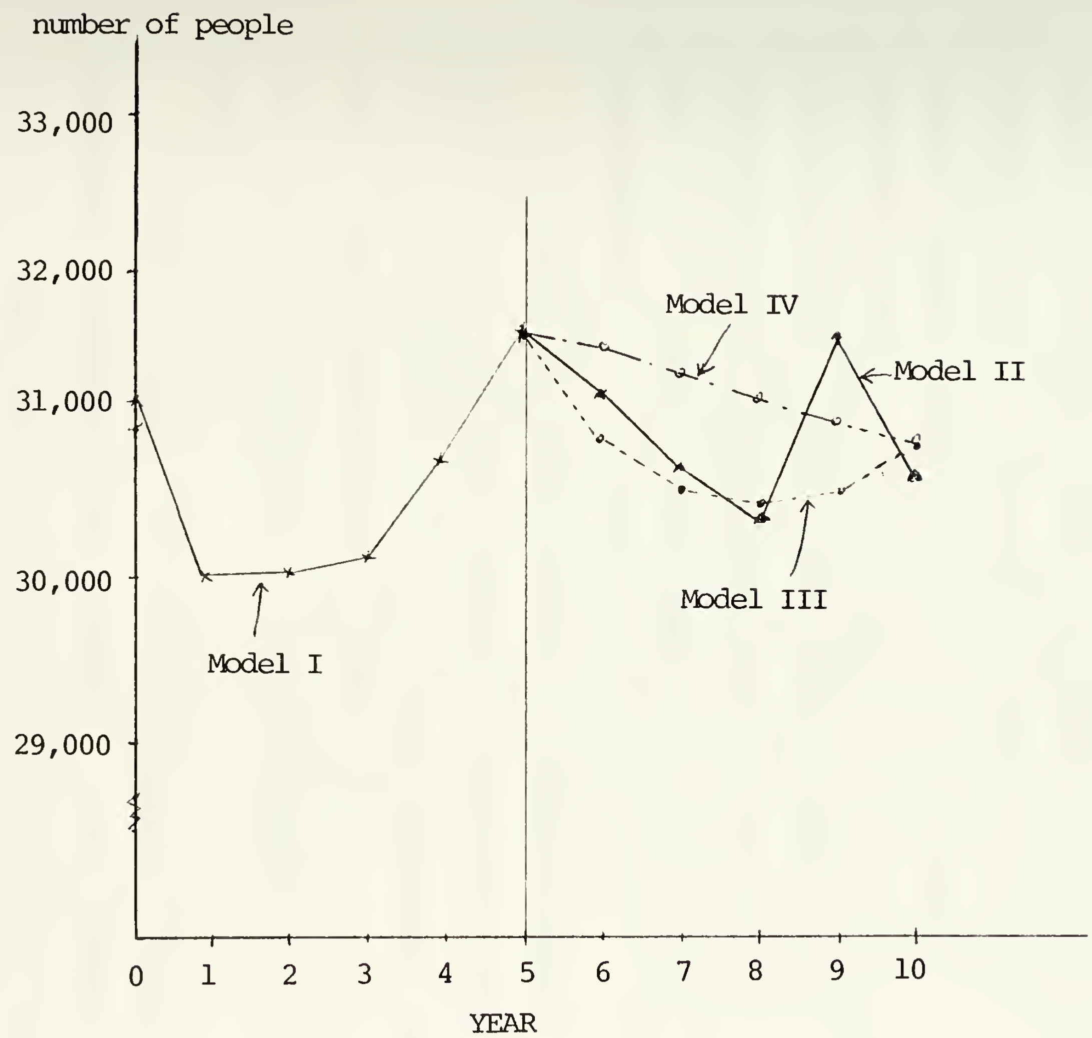


FIGURE 29. Movement of Total Force Size (New Policy)

In the steady state period when we leave out this corporal's constraint we have the size of force structure decreasing rapidly but stopping at a little above the minimum manpower requirement (i.e. 30,000), except in Model IV it is still decreasing (in this case it stops at 24,016 at year 100!). When another 5 year planning period is made the size of the force is increasing again by applying Model II and III but does not exceed the maximum size as it happens in year 5, and also the rate of increase is relatively smaller than in the first 5 year period. If the manpower planner is not satisfied with this solution, they can try another policy, for example suppose they change the promotion and attrition policy with this following condition: (See Table 8, Table 9 and Table 10), and keep the other constraints as in the previous values.

The result of using these new policies toward the solution is shown in Figure 29. Notice that in the transition period the size of force structure is also increasing, but relatively smaller than the previous solution, and in steady state period the size of the force does not vary as much as in the other models.

It seems further, that if we leave the corporal's rank proportion constraint out of Model I, the solution might be better since the size of the force moved not necessarily increase at the end of year 5 and thus the force size alterations are smoother. However all of these conditions are

TABLE 8. New Promotion Policy (Constraints)

Lower Bound	$Z_{i,t}$ or $X_{i,t}$	Upper Bound	With Respect to
0.4	$Z_{1,t}$	0.6	$S_{1,t}$
0.3	$X_{2,t}$	0.5	$S_{2,t}$
0.3	$Z_{3,t}$	0.5	$S_{3,t}$
0.4	$Z_{4,t}$	0.6	$S_{4,t}$
0.4	$Z_{5,t}$	0.6	$S_{5,t}$
0.3	$X_{3,t}$	0.5	$S_{6,t}$
0.3	$Z_{7,t}$	0.5	$S_{7,t}$
0.2	$X_{4,t}$	0.4	$S_{8,t}$
0.4	$Z_{9,t}$	0.6	$S_{9,t}$
0.4	$Z_{10,t}$	0.6	$S_{10,t}$
0.2	$Z_{11,t}$	0.4	$S_{11,t}$
0.2	$X_{6,t}$	0.4	$S_{12,t}$
0.15	$Z_{13,t}$	0.3	$S_{13,t}$
0.	$Z_{14,t}$	0.2	$S_{14,t}$

TABLE 9. New Attrition Policy (Constraints)

Lower Bound	$a_{i,t}$	Upper Bound	With Respect to
0	$a_{1,t}$	0.01	$S_{1,t}$
0	$a_{2,t}$	0.02	$S_{2,t}$
0	$a_{3,t}$	0.03	$S_{3,t}$
0	$a_{4,t}$	0.04	$S_{4,t}$
0	$a_{5,t}$	0.08	$S_{5,t}$
0	$a_{6,t}$	0.1	$S_{6,t}$
0	$a_{7,t}$	0.1	$S_{7,t}$
0	$a_{8,t}$	0.2	$S_{8,t}$
0	$a_{9,t}$	0.2	$S_{9,t}$
0	$a_{10,t}$	0.2	$S_{10,t}$
0	$a_{11,t}$	0.2	$S_{11,t}$
0	$a_{12,t}$	0.2	$S_{12,t}$
0	$a_{13,t}$	0.2	$S_{13,t}$
0	$a_{14,t}$	0.2	$S_{14,t}$

TABLE 10. New Non-Zero Elements of Flow Matrix Q

$q_{j,i}$		$q_{i,i}$	
$q_{2,1}$	0.4	$q_{1,1}$	0.6
$q_{3,2}$	0.285	$q_{2,2}$	0.715
$q_{4,3}$	0.344	$q_{3,3}$	0.63
$q_{5,4}$	0.405	$q_{4,4}$	0.563
$q_{6,5}$	0.420	$q_{5,5}$	0.5
$q_{7,6}$	0.388	$q_{6,6}$	0.512
$q_{8,7}$	0.383	$q_{7,7}$	0.517
$q_{9,8}$	0.325	$q_{8,8}$	0.475
$q_{10,9}$	0.444	$q_{9,9}$	0.356
$q_{11,10}$	0.400	$q_{10,10}$	0.4
$q_{12,11}$	0.400	$q_{11,11}$	0.4
$q_{13,12}$	0.230	$q_{12,12}$	0.64
$q_{14,13}$	0.180	$q_{13,13}$	0.692
$q_{15,14}$	0.124	$q_{14,14}$	0.716

Note: These values are also obtained from the solution of Model III

$f_{0,1} = 1610$ $f_{0,10} = 50$ $f_{0,11} = 80$

dependent upon the higher authority, that is, which solution they might think will be the best for the Air Force at that moment.

For the manpower planner, it is just a matter of adjusting the policies, i.e. constraints, of the model, in order to achieve a desired solution.

The advantage of these models over the present approach is that using the models lets us analyze the interactions among the various policies and predict the results of these policies before they are used. The result should be policies which are more consistent with the goals of the Air Force.

V. SUMMARY

Creating a well-balanced force structure from the out of balance situation, and then maintaining its balanced condition for another future period is the primary goal of this study. Based on a basic flow equation from an ordinary manpower planning model [4], a linear programming model is developed, using this basic flow equation as its main constraint along with stock, promotion and attrition, and budget constraints.

For creating a well-balanced force structure, a Model I whose approach is a linear programming method is used, and a period of 5 year-planning program is chosen for its transition period. Following the solution from Model I, three different types of models are applicable for maintaining the balanced condition. For a long run planning horizon (i.e. 5 years), Model II is used, while for short run planning (i.e. annually), Model III is used (in fact Model III can also be used for long run planning, by solving the problem repeatedly). In addition to the linear programming models, a cross-sectional Model IV [1] is also used for both short and long range solutions.

Some examples of the application of the models are presented in this paper. Through these various examples, we can observe how each model solved the corresponding problem. The choice of which model is to be used depends upon the

situation of the problem and what goals the manpower planner wants to achieve. What is apparent is that all of the models are very sensitive to the policy constraints, hence by making a slight adjustment of these constraints, the manpower planner can easily modify the solution. These three linear programming models used in Model I, II and III can be solved by MPS-360, a mathematical programming package which is available in IBM-360 computer. For Model IV we can solve either by computer or by hand (if computer is not available).

By using these models, the manpower planner can save a considerable amount of time and energy in preparing a manpower plan for the Indonesian Air Force in the future.

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